

Linearized Optimal Power Flow and Distribution for Marginal Prices in Distribution Networks

P Srividya Devi¹, M. Rekha²

¹Associate Professor, ²Assistant Professor
Electrical and Electronics Engineering,
GRIET, Hyderabad, India

Abstract

The increasing integration of distributed energy resources (DERs), including solar photovoltaic systems, electric vehicles and battery storage, is transforming conventional passive distribution networks into active distribution networks. This transformation introduces bidirectional power flow, voltage regulation, congestion management and economic-dispatch challenges. Conventional AC Optimal Power Flow (ACOPF) models accurately represent these effects but are nonlinear and nonconvex, which can make repeated real-time computation difficult. This project presents a loss-sensitivity-based linearized Optimal Power Flow (OPF) framework for balanced distribution networks and applies it to Distribution Locational Marginal Pricing (DLMP). The approach linearizes the power-flow and loss relationships using suitable approximations and loss-sensitivity factors, while modelling reactive demand through load power factor. A MATLAB-based implementation demonstrates a merit-order market clearing stage followed by sensitivity-based nodal price calculation for radial and meshed six-bus configurations. The study shows that power factor and network topology materially affect DLMP values. Improving load power factor reduces reactive demand, losses and marginal prices, while meshed operation can improve power sharing and market surplus. The project further reports validation against ACOPF on larger distribution systems and highlights the computational advantage of linearization for scalable active-distribution-network operation.

Keywords: Active Distribution Network, Optimal Power Flow, Linearized OPF, Distribution Marginal Pricing, Loss Sensitivity, Distributed Energy Resources, Power Factor, MATLAB.

1. Introduction

The rapid integration of distributed energy resources has changed the operating characteristics of distribution systems. Solar photovoltaic generation, electric vehicles and battery storage can produce power locally and can cause bidirectional power flow. Consequently, distribution-system operators must manage voltage limits, network congestion, losses and the economic dispatch of DERs. Distribution Locational Marginal Pricing provides a mechanism for representing the marginal economic value of supplying power at individual network buses.

Unlike transmission systems, distribution networks commonly have relatively high R/X ratios and stronger voltage sensitivity. Therefore, simplified transmission-oriented DCOPF formulations are less accurate for distribution applications. The supplied project identifies linearized OPF as an attractive alternative because it can retain the important distribution characteristics while reducing computational complexity.

Problem Statement and Objectives

The project addresses the need for a purely linearized OPF framework that can be applied to both radial and meshed balanced distribution networks, represent losses using sensitivity factors, incorporate the relationship between active and reactive load demand through power factor, and derive economically meaningful locational prices. The framework is also intended to remain computationally efficient and scalable as the distribution network becomes large.

1. Formulate a linear OPF model that captures the high R/X characteristics of distribution lines and is applicable to generic network topologies.
2. Incorporate a power-factor-based load model that links reactive power demand to active power consumption.
3. Derive locational marginal prices for generators and loads, with load prices reflecting their power-factor characteristics.
4. Evaluate accuracy, computational efficiency and scalability by comparing the linearized formulation with conventional ACOPF results.

The study focuses on balanced active distribution networks and considers radial and weakly meshed configurations. The implementation described in the project uses a six-bus test system for the MATLAB demonstration and discusses validation on 33-bus, 69-bus and larger distribution systems.

2. Methodology

The proposed methodology uses a linearized AC Optimal Power Flow model to calculate Distribution Locational Marginal Prices. The nonlinear AC power-flow relationships are simplified using small-angle approximations and loss-sensitivity factors obtained from an initial operating point. Load reactive requirements are represented using active power and power factor. The resulting linearized OPF is solved in MATLAB, after which the voltage and loss estimates can be updated iteratively to improve accuracy. Finally, the linearized results are compared with conventional ACOPF results. The proposed methodology begins with the collection and input of the bus, branch, generation, and load data of the distribution network. An initial power-flow analysis is then performed to determine the base-case voltage magnitudes, voltage angles, and power losses. These base operating conditions are used to linearize the nonlinear power-flow and loss equations using appropriate loss-sensitivity factors. The resulting linearized Optimal Power Flow (OPF) model is solved to determine the optimal generation dispatch, followed by merit-order market clearing to satisfy the system demand. Based on the obtained operating conditions and sensitivity factors, bus-wise Distribution Locational Marginal Prices (DLMPs) are calculated by incorporating the effects of load power factor and associated reactive power requirements. Finally, the voltage estimates and loss coefficients are updated iteratively, and the results obtained from the proposed linearized OPF are validated against conventional AC Optimal Power Flow (ACOPF) results in terms of voltage profiles, power flows, losses, and pricing signals

The MATLAB demonstration uses a two-stage procedure. First, generator offers are arranged in merit order and allocated until the total active-power demand is satisfied. Second, a sensitivity-based correction is applied to the base clearing price to obtain locational prices at the load buses for radial and meshed cases. The implementation uses bus-level loss-sensitivity coefficients and several load power-factor scenarios.

The project models load reactive power through a tangent factor T_f , so that reactive demand is linked to active power demand. The load DLMP relationship presented in the source is:

$$\lambda_{D,n,i} = \lambda_{PG,n} + T_{f,i} \cdot \lambda_{QG,n} \dots \dots \dots (1)$$

This formulation makes the price paid by a load dependent on its reactive-power characteristic. As the power factor improves, reactive demand is reduced, which reduces the associated marginal loss effect and can lower the DLMP.

- 6-bus radial and weakly meshed distribution system.
- 33-bus radial distribution system.
- 69-bus radial and weakly meshed distribution systems.
- Large-scale distribution feeders with more than 1000 buses; the source reports scalability testing up to 2241 buses.

MATLAB is used as the primary computational environment for modelling, simulation, matrix calculations, iterative updates and visualization. The source document notes that MATLAB supports optimization routines and can be used to generate voltage, power-flow, convergence and pricing plots. The supplied MATLAB demonstration is deliberately simplified. The sensitivity coefficients used in the prototype are illustrative rather than being fully recomputed from the formal linearized power-flow matrices. The merit-order allocation stage also does not explicitly enforce all line and voltage constraints, whereas the full OPF formulation does. The source further identifies iterative updates as a way to improve the quantitative accuracy of the linearized solution.

3. System Design

The six-bus balanced distribution network used for the MATLAB demonstration contains the grid at Bus 1, load buses and distributed generators at selected buses. The system is examined in both radial and meshed configurations. Loads D1–D5 are placed at buses 2–6, while the grid and three DERs submit generation offers at buses 1, 4, 5 and 6.

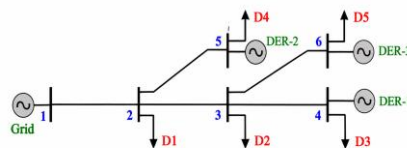


Figure 1. DER-enhanced six-bus distribution-system topology used in the project.

Table 1.1: Assumptions and Generation data offered by the DER

Bus No.	Generator ID	Offered MW	Offered Price (\$/MWh)
1	Grid	10	2
4	DER-1	0.5	1
5	DER-2	2.5	3
6	DER-3	0.5	1

System Data and Assumptions

- The main demonstration uses a balanced six-bus distribution system.
- Five loads are located at buses 2–6 and the reported total active-power demand is 2.65 MW.
- The system is studied in radial and meshed configurations to examine topology-dependent pricing.
- Load power factors are varied to demonstrate the effect of reactive demand on DLMP.
- The linearized formulation uses loss-sensitivity information derived from a base operating point.
- The project evaluates both economic outcomes and electrical accuracy against nonlinear ACOPF results.

The functional architecture of the proposed system can be summarized as a sequence from network data acquisition through power-flow linearization, OPF solution, DLMP calculation and validation. Loss-Sensitivity-Based Pricing

Loss sensitivity factors are used to represent how changes in active and reactive power affect network losses. These sensitivities provide the linear correction needed to move from a common market clearing price to bus-specific DLMPs. The approach is intended to preserve the economically important effects of losses and congestion without the full computational burden of repeatedly solving a nonlinear nonconvex ACOPF.

4. RESULTS AND DISCUSSION

The simulation results in the supplied document demonstrate the influence of both load power factor and network topology on DLMP and market performance. MATLAB is used for the calculations and for plotting the resulting price trends.

The radial-network plot shows a consistent decrease in LMP as load power factor improves from 0.80 to 0.98. Improved power factor reduces reactive power demand, which in turn reduces network losses and the marginal cost of supplying power. Buses farther from the source show somewhat higher LMP values because of increased loss sensitivity.

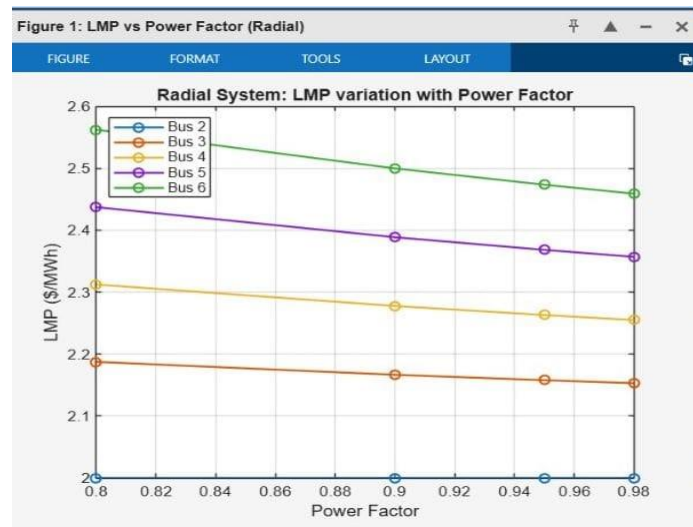


Figure 2. LMP variation with load power factor in the radial network.

Radial versus Meshed Market Performance

The market-pricing comparison indicates similar total-revenue and total-cost trends for radial and meshed systems, while the meshed configuration shows a marginally higher market surplus. The project attributes this to improved power-sharing capability and reduced losses in meshed operation.



Figure 3. Market-pricing comparison between radial and meshed configurations.

Key Discussion Points

- Power factors have a direct influence on DLMP: lower lagging power factors result in greater reactive demand and higher marginal prices.
- Improving power factor reduces reactive requirements and the loss-related component of the nodal price.
- Network topology changes the spatial distribution of prices because meshed networks provide additional paths for power sharing.
- Buses electrically farther from the source can experience higher prices because of larger losses and sensitivity effects.

- The close relationship between cost and revenue in the market comparison supports the internal consistency of the pricing mechanism.
- The linearized approach provides faster and more interpretable calculations than a full nonlinear formulation, while retaining the major qualitative pricing behaviour.

Accuracy and Scalability

The supplied project reports a maximum voltage error of 0.0345% when the proposed approach is compared with nonlinear ACOPF. It also reports validation on 33-bus radial, 69-bus radial and 69-bus weakly meshed systems and scalability tests up to 2241 buses. The source states that the computational-time advantage of the linearized approach becomes more significant as system size increases.

- The MATLAB demonstration uses simplified, illustrative sensitivity coefficients rather than fully recomputing the formal sensitivity matrices.
- The merit-order dispatch demonstration does not explicitly enforce every voltage and line-flow constraint during allocation.
- A single linear correction is used in the simplified implementation, whereas the full framework can iteratively update voltages and loss sensitivities.
- The current research scope is focused on balanced networks; unbalanced three-phase operation is identified as future work.

5. CONCLUSION

This project develops and demonstrates a loss-sensitivity-based linearized Optimal Power Flow framework for balanced active distribution networks. The formulation represents reactive load consumption as a function of active power and power factor and linearizes active and reactive power-flow losses using sensitivity factors obtained from base-case system states. The resulting framework provides locational marginal prices for generators and different load characteristics.

The six-bus case study demonstrates the influence of network topology and load power factor on DLMP. Radial-network prices decrease as power factors improve, while meshed operation can reduce congestion and losses and provide a slightly higher market surplus. The source document further reports close agreement with ACOPF solutions on larger radial and weakly meshed systems and a growing computational advantage as system size increases.

Overall, the study indicates that linearized OPF with loss sensitivity factors can provide a practical and computationally efficient basis for DLMP calculation and active distribution-network operation. The next stages identified in the project include extension to unbalanced three-phase networks, treatment of renewable-generation and load uncertainty, decentralized solution methods, and real-time market implementation.

- Extend the framework to unbalanced three-phase distribution systems and different connection types such as wye and delta.
- Integrate forecasting, stochastic optimization or robust optimization for renewable-generation and load uncertainty.
- Develop distributed or decentralized algorithms for very large networks with many prosumers.

- Integrate day-ahead market clearing with real-time balancing and shorter operational intervals.
- Explore interaction with local energy and flexibility markets.

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