International Journal on Science and Technology (IJSAT)
E-ISSN: 2229-7677 ● Website: www.ijsat.org ● Email: editor@ijsat.org

Electron Trajectory of Linearly Polarized Undulator in THz Region

Mukesh Kumar[1], Kuldeep Kumar[2], Mukesh Jewariya[2]

[1]University School of Basic and Applied Sciences, GGSIP University, Delhi
[2]National Physical Laboratory, Dr. Radhakrishnan Marg, Delhi

Abstract

THz-FEL has unprecedented advantages over quantum lasers in comparison to their spectral brightness and wide tunability. Electron beam trajectory is responsible for the study of spectral brightness and wide tunability in a planar undulator. Electron trajectories are studied in planar undulator using relativistic Lorentz force equation which leads the derivation of trajectory. Trajectory of electron beam is worked out numerically using Runge Kutta 4th order method. Trajectories are visualized using various planar undulator parameters and linear accelerator parameters for THz domain.

Keywords: THz, FEL, Planar Undulator, Relativistic Lorentz Force Equation.

1. Introduction

The terahertz (THz) band defined as the frequency range from about 0.1 THz to 10 THz has designated the “THz gap” because it lies in electromagnetic spectrum between microwave and Infrared region [1]. Comprehensive overview of THz science and technology is presented and discussed by Gallerano [2]. Over two decades this gap has narrowed significantly and sources operated by linear accelerators, which generates relativistic electron bunch is passed through a spatially varying periodic magnetic structure called as planar undulator have emerged as most versatile method to produce intense, tunable and highly monochromatic THz source [3,4]. Planar undulator is widely used undulator configuration for THz generation due to its comparatively simple magnet design, well established design theory and compatibility [4]. In planar undulator, two arrays of magnets confine the electron beam to a transverse plane enforcing the electron oscillations in this plane. These oscillations are known as transverse oscillations responsible for emission of radiation depending on the energy and current of electron beam. The frequency bandwidth, polarization purity and angular distribution of the emitted THz radiations are all direct consequences of the transverse and longitudinal oscillations of electrons inside the spatially varying periodic magnetic field of planar undulator [5]. The departure of the real trajectory from the ideal sinusoidal path, is attributed to magnetic field errors, space-charge forces and beam-energy spread, propagates directly into a degradation of the THz output [3,6]. Spatial confinement and control of electron trajectory through the planar undulator is fundamental problem for design of next generation THz radiation sources.

The equation of motion for an electron in an ideal and purely sinusoidal planar undulator field yields a closed form harmonic trajectory; the transverse velocity oscillates at the undulator period, while the longitudinal velocity acquires a characteristic second harmonic modulation that gives rise to the well-known undulator radiation formula relating wavelength, undulator period, electron beam current, electron energy and the undulator strength parameter K [7]. The ideal treatment of planar undulator is the basic principle of most design level calculations and simulations of THz undulator sources. Real planar undulator magnetic fields, on the contrary, fall off hyperbolically away from the central plane rather than remaining perfectly sinusoidal on-axis. Off-axis field profile offers a betatron oscillation on top of the nominal undulating motion and his additional motion is naturally described using Mathieu functions. Their multiparticle, time-domain extension of this trajectory model demonstrates that betatron motion measurably reshapes the coherence and spectral purity of the emitted THz radiation, an effect that is easy to overlook in frequency domain treatments but becomes significant for the short, low-energy bunches typical of compact THz facilities [5]. In this work, electron trajectories are investigated analytically using Generalised Bessel Functions (GBFs) through planar undulator. Relativistic Lorentz Force equation is employed for the calculations of electron trajectories when electron beam with current I_b is undergoes oscillations in the plane of undulator [8].

Field of Planar Undulator

The magnetic field of linearly polarized undulator is given by, $\vec{B} = 0.\hat{x} + B_0 \sin(\omega_u t)\hat{y} + 0.\hat{z}$, where B_0 is the amplitude of the undulator field. The formula required for the amplitude of undulator field B_0 produced by Halbach configuration with Pure Permanent Magnet (PPM) is given by

$$B_0 = 2\varepsilon B_{rem}(1-e^{-k_u L'})(e^{-k_u g_u/2})\sin c(\varepsilon\pi/M) \quad (1)$$

where B_{rem} is the remnant field of pure permanent magnet and B_{rem}=1.1 Tesla in case of NdFeB [9]. In our study, we choose $\varepsilon = 1$ and M=4 which represents number of magnets required to complete one period, $L' = \lambda_u/4$. Using equation (1), we obtain the peak value of undulator field which is given by

$$B_0 = 1.57(\text{Tesla})\exp(-\pi g_u/\lambda_u) \quad (2)$$

The expression of peak value of undulator parameter K is given by

$$K = 0.148\lambda_u(\text{mm})\exp(-\pi g_u/\lambda_u) \quad (3)$$

where g_u is undulator gap in mm. We carry out the analysis of practically designed parameters of undulator to produce THz radiations in the next section. Undulator parameter given by equation (3) describes the control of undulator gap and undulator period. It is noted that optimization of undulator gap and period must be achieved for maximum coupling of electron beam energy to THz radiation.

Electron Trajectories

On the application of the Relativistic Lorentz Force (RLF) equation $d\vec{p}/dt = -ec(\vec{\beta} \times \vec{B})$ for the calculations of velocity and electron trajectories using Generalised Bessel Function (GBF), the 3D trajectory through planar undulator is written as

$$\vec{r}(t) = -\frac{Kc}{\gamma\omega_u}\sin(\omega_u t).\hat{x} + 0.\hat{y} + \left[\beta^* ct - \frac{K^2 c}{8\gamma^2\omega_u}\sin(2\omega_u t)\right].\hat{z} \tag{4}$$

where $\beta^* = 1 - \frac{1}{2\gamma^2}(1 + K^2/2 + (\gamma\psi)^2)$ is average velocity of electron beam. (5)

Equation (4) describes the electron trajectory with axial and longitudinal oscillations. Longitudinal oscillations take place at the double of the fundamental frequency which specifies the fact that longitudinal oscillation has zero contribution in the emission of THz radiations, only axial oscillations are responsible for the emission of radiations. If N_e is the number of electrons emitted from the photoinjector and fed to the planar undulator of length L_u, the current of electron beam is I_b, the trajectory of electrons will be [10]

$$\vec{r}(t) = -(KI_b L_u/e\gamma\omega_u)\sin(\omega_u t).\hat{x} + 0.\hat{y} + (I_b L_u/e)\left[\beta^* t - \frac{K^2}{8\gamma^2\omega_u}\sin(2\omega_u t)\right].\hat{z} \tag{6}$$

Where e is the electronic charge, I_b is electron beam current, L_u is undulator length. It is evident that maximum amplitude of transverse oscillation is multiplied than single electron oscillations. Longitudinal oscillations are also modified but oscillates at the second harmonic frequency. Amplitude of electron oscillations in undulator plane is enhanced on increasing electron beam current I_b. γ is electron beam energy factor.

Results and Discussion

In this work, electron beam parameters are used for calculations of electron beam trajectories in planar undulator [11]. Undulator parameters for THz domain are employed for simulation and calculations of electron beam trajectories [12]. For simulations in THz domain, Runge Kutta Method of 4th order equations are employed to calculate electron beam trajectories for electron beam and planar undulator parameters.

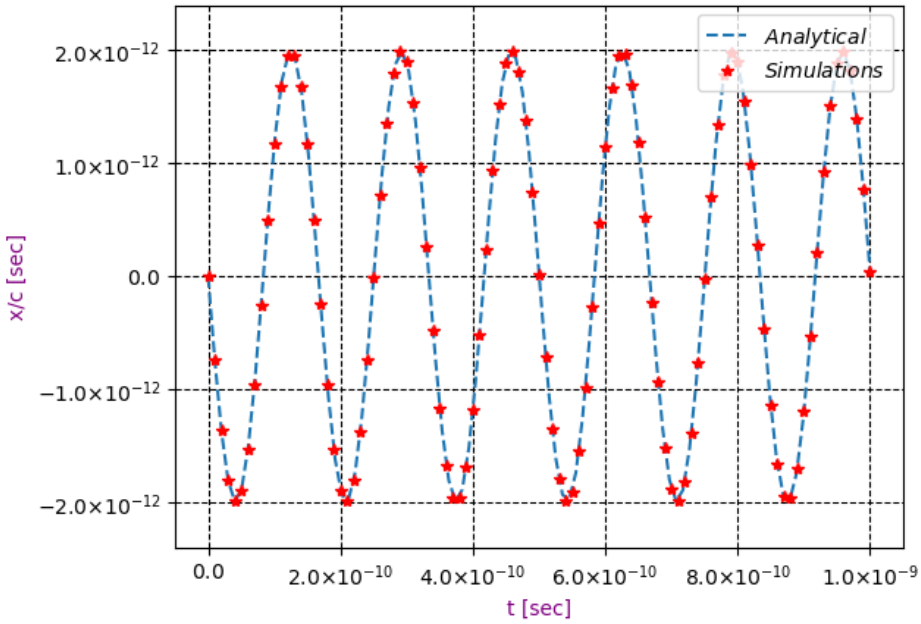

Fig.1 This figure displays the single electron trajectory for the undulator gap g_u=2.5 cm, undulator period λ_u=5 cm, and undulator parameter K=1.54.

Fig.1. visualizes the electron beam trajectory for planar undulator and linear accelerator parameters. The displayed trajectory is consistent with equation number (4). Fig.1. displays the transverse oscillations confined in x-z plane. The maximum amplitude of oscillations is from $2x10^{-12}$ to $-2x10^{-12}$ for planar undulator with standard parameters. The analytical and numerical trajectories are overlapped completely. Numerical trajectories are visualized using Runge Kutta Method of 4th order. Thus, electron oscillations through planar undulator confined in x-z plane depending on various parameters are compatible with analytical and numerical methods. Numerical trajectory overlaps completely with analytical trajectory for given undulator and accelerator parameters.

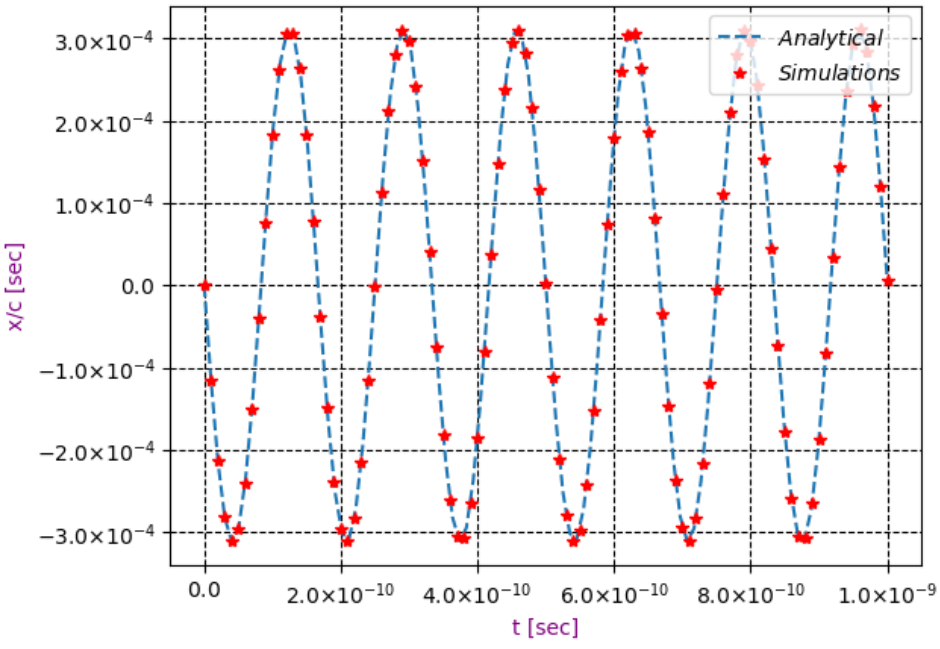

Fig.2 This figure displays the electron beam trajectory for the undulator gap g_u=2.5 cm, undulator period λ_u=5 cm, electron beam energy E_b=10 MeV, beam current I_b=10 mA and undulator parameter K=1.54.

Fig.1. reveals the single electron trajectory having undulator gap g_u=2.5 cm, undulator period λ_u=5 cm, K=1.54, electron beam energy E_b=10 MeV and amplitude of undulator field B_0=0.33 Tesla. It is evident that the Runge Kutta Method and analytical method both produce trajectory of electron in planar undulator is overlapped completely. Fig.2. shows the electron trajectory produced for the various parameters. The electrons beam current I_b=10 mA, beam energy E_b=10 MeV, energy factor γ=20.53, undulator field B_0=0.33 Tesla, undulator gap g_u=2.5 cm, undulator period λ_u=5 cm and K=1.54 display that both analytical and Runge Kutta (RK-4) method produce electron trajectory which fits perfectly. The results of trajectories by both methods overlap perfectly. The amplitude of trajectory for given parameters is $3x10^{-4}$ which behaves as sine wave confined in x-z plane. However, the amplitude of trajectory corresponding to 20 mA electron beam current I_b has increased appreciably which is consistent to equation number (6). The amplitude of transverse oscillations confined in x-z plane is responsible for energy transfer from electron beam to electromagnetic radiation depending on the electron beam trajectory through undulator. The maximum amplitude of oscillations in x-z plane is from $-3x10^{-4}$ to $3x10^{-4}$ for electron beam current I_b =10 mA.

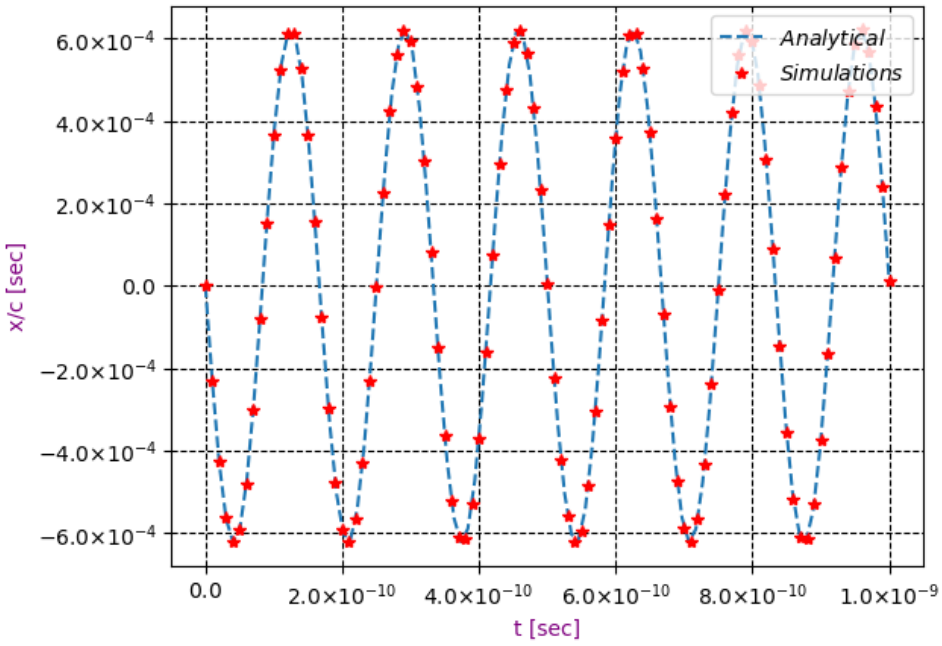

Fig.3 This figure displays the electron beam trajectory for the undulator gap g_u=2.5 cm, undulator period λ_u=5 cm, electron beam energy E_b=10 MeV, beam current I_b=20 mA and undulator parameter K=1.54.

Fig.3. demonstrates the electron trajectory for electron beam current I_b=20 mA. The maximum amplitude of transverse oscillations is from $-6x10^{-4}$ to $6x10^{4}$ which is consistent with equation (6). Both the trajectories overlap with analytical and numerical methods perfectly. Fig.4. has maximum amplitude of transverse oscillations from $-1x10^{-3}$ to $1x10^{3}$ for electron beam current I_b=30 mA. Both method displays perfect fit of electron beam trajectories. Fig.5. reveals the electron trajectory for electron beam current I_b=40 mA for given electron beam energy. Both the methods corresponding to analytical and numerical trajectories overlap completely. This shows that numerical and analytical methods work hand in hand.

Fig.6 This figure displays the electron beam trajectory for the undulator gap g_u=2.6 cm, undulator period λ_u=5 cm, electron beam energy E_b=10 MeV, beam current I_b=10 mA and undulator parameter K=1.45. It is evident that for undulator gap g_u=2.6 cm, it reduces substantial amount of undulator field and undulator parameter K=1.45 which in turn reduces the amplitude of oscillations.

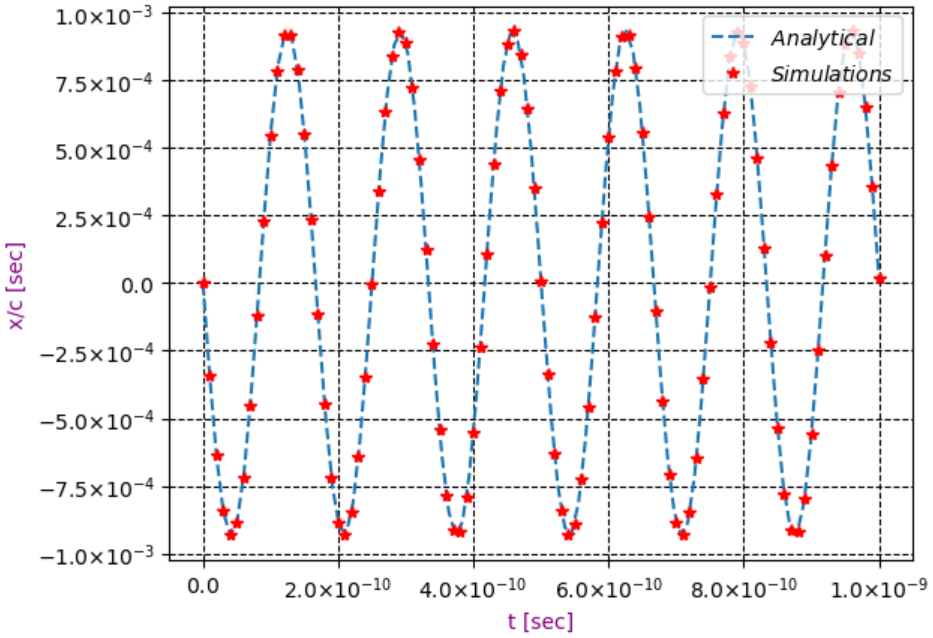

Fig.4 This figure displays the electron beam trajectory for the undulator gap g_u=2.5 cm, undulator period λ_u=5 cm, electron beam energy E_b=10 MeV, beam current I_b=30 mA and undulator parameter K=1.54.

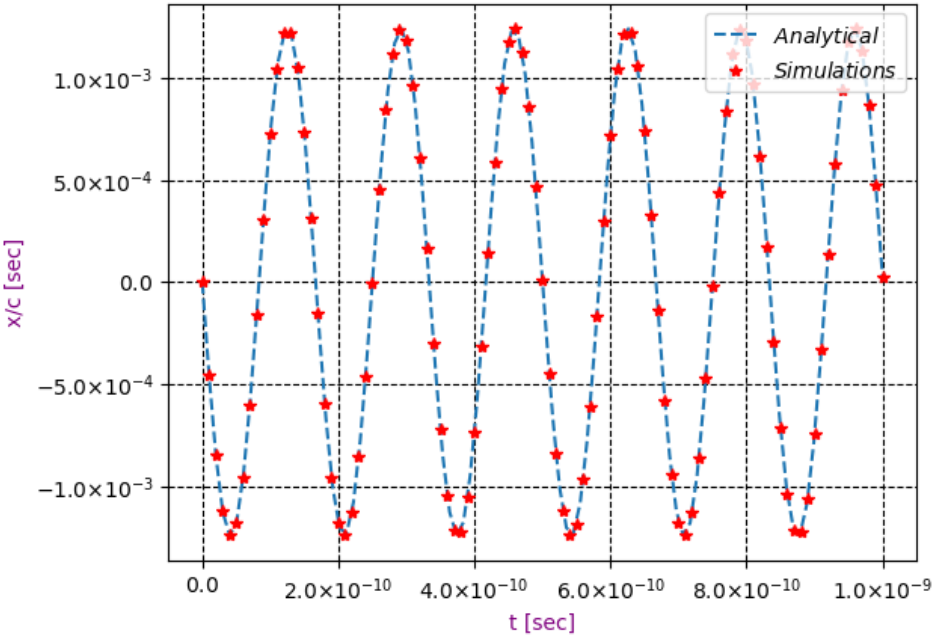

Fig.5 This figure displays the electron beam trajectory for the undulator gap g_u=2.5 cm, undulator period λ_u=5 cm, electron beam energy E_b=10 MeV, beam current I_b=40 mA and undulator parameter K=1.54.

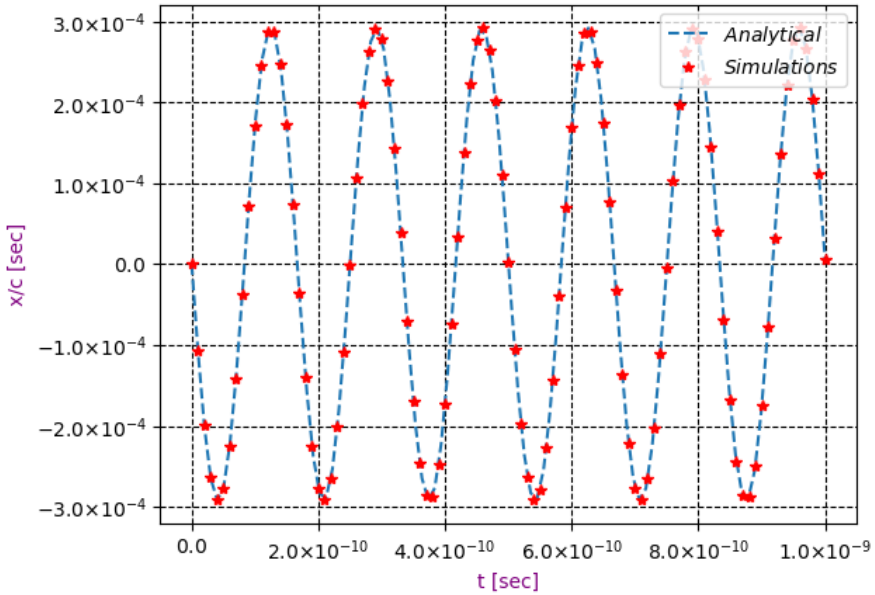

Fig.6 This figure displays the electron beam trajectory for the undulator gap g_u=2.6 cm, undulator period λ_u=5 cm, electron beam energy E_b=10 MeV, beam current I_b=10 mA and undulator parameter K=1.45.

Fig.7 This figure displays the electron beam trajectory for the undulator gap g_u=2.7 cm, undulator period λ_u=5 cm, electron beam energy E_b=10 MeV, beam current I_b=10 mA and undulator parameter K=1.36. In fig.7. electron trajectories are visualized for numerical and analytical methods. It is evident that when undulator period is decreased upto g_u=2.7 cm, the amplitude of transverse oscillations is reduced appreciably which is consistent with equation (6) which is good for efficient operation of THz-FEL because for efficient operation the electron oscillations must be confined in x-z plane and should be near to undulator axis of Free Electron Laser (FEL). The spectral brightness will be higher if the electron oscillations are near to the undulator axis. Fig.8 describes the behaviour of electron trajectory for analytical and numerical methods with standard undulator and accelerator parameters. The amplitude of oscillations is from $-2x10^{-4}$ to $2x10^{4}$ corresponding to THz domain.

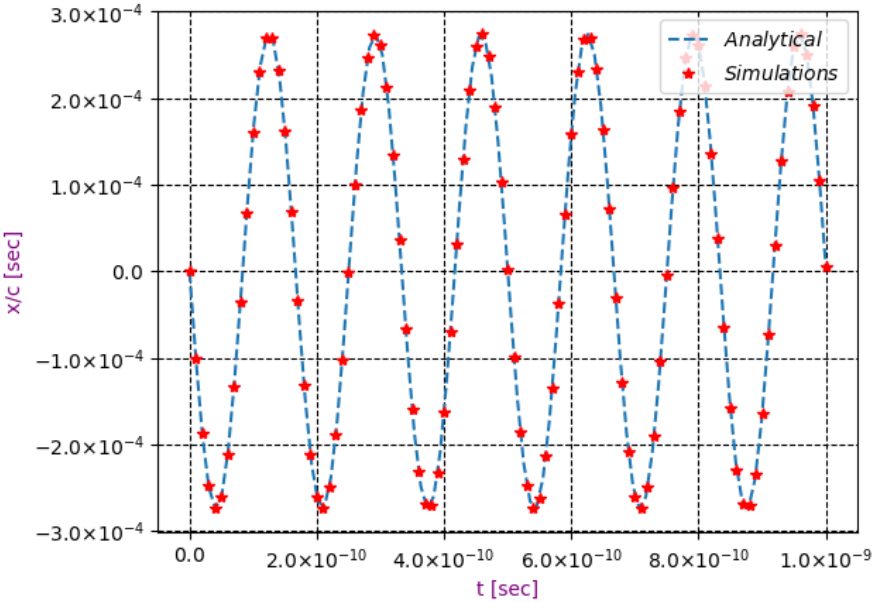

Fig.7 This figure displays the electron beam trajectory for the undulator gap g_u=2.7 cm, undulator period λ_u=5 cm, electron beam energy E_b=10 MeV, beam current I_b=10 mA and undulator parameter K=1.36.

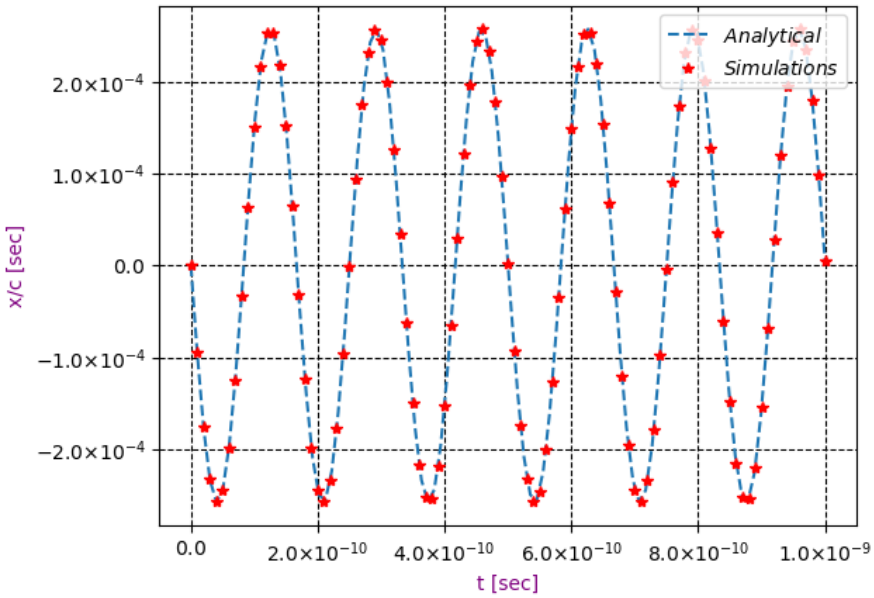

Fig.8 This figure displays the electron beam trajectory for the undulator gap g_u=2.8 cm, undulator period λ_u=5 cm, electron beam energy E_b=10 MeV, beam current I_b=10 mA and undulator parameter K=1.28.

Conclusion

In this work, the trajectories of electron beam having current I_b is discussed and visualized using Runge Kutta Method of 4th order with analytical method in THz region. Numerical Electron trajectories are perfectly fits with analytical methods. The study of electron trajectories is useful for designing potential THz-FEL systems with planar undulator and linear accelerator in THz region.

Acknowledgement

This work is supported by the University School of Basic and Applied Sciences (USBAS), Guru Gobind Singh Indraprastha University, New Delhi.

References

1. T. Ping, H. Jiang, L.K. Feng, X.Y. Qian, F.M. Wu, Terahertz radiation sources based on free electron lasers and their applications, Science China Information Sciences, 55(1) 1-15 (2012)
2. G.P. Gallerano, S. Biedron, Overview of Terahertz Radiation Sources, Proceedings of the 2004 FEL Conference, 216-221 (2004)
3. S. Krainara, H. Zen, S. Chatani, T. Kii, H. Ohgaki, Properties of THz coherent undulator radiation generated from a compact accelerator source at Kyoto University. Review of Scientific Instruments, 90(10), 103307 (2019) https://doi.org/10.1063/1.5110342

4. S. Rimjaem, et al. Development of a compact electromagnetic undulator for linac-based coherent THz radiation source in Thailand. Nuclear Instruments and Methods in Physics Research A (2018). https:www.sciencedirect.com/science/article/abs/pii/S0168900218307484
5. V. Joshi, S. Ghosh, Multiparticle time-domain analysis of coherent undulator radiation, Physical Review Accelerators and Beams, 22(2) 020702 (2019) https://doi.org/10.1103/PhysRevAccelBeams.22.020702
6. A. Fisher, J. Jin, P. Musumeci, Controlling beam trajectory and transport in a tapered helical undulator, Journal of Synchrotron Radiation, 32(2), 331–339 (2025) https://doi.org/10.1107/S1600577524012463
7. Q. Jia, Harmonic Motion of Electron Trajectory in Planar Undulator, ProceedingsofPAC09, Vancouver, BC, Canada, WE5RFP088, Light Sources and FELs T15-UndulatorsandWigglers, https://s3.cern.ch/inspire-prod-files-f/ff5bef909a182ecaf8b724491d0bf958
8. G. Dattoli, L. Giannessi, L. Mezi, M. Richetta, and A. Torre, Linear undulator brightness: Inclusion of sextupolar magnetic-field contributions and of higher-order energy corrections, Physical Review A, 45(6) 4023-4035 (1992)
9. C. A. Brau, Free-electron laser, Academic Press, San Diego, 1990
10. D. Attwood, A. Sakdinawat, X-Rays and Extreme Ultraviolet Radiation: Principles and Applications, Cambridge University Press, 1st Edition (2016)
11. Phillip Sprangle, Joseph Penano, Bahman Hafizi, Daniel Gordon, Steven Gold, Antonio Ting, Chad Mitchell, High average current electron guns for high-power free electron lasers, Physical Review Special Topics- Accelerators and Beams 14, 020702 (2011)