International Journal on Science and Technology (IJSAT)

E-ISSN: 2229-7677 ● Website: www.ijsat.org ● Email: editor@ijsat.org

Nilpotent matrix and Hermitian matrix

M Vinothkumar Muniyandi

Abstract:

Nilpotent and Hermitian matrices are two important classes of matrices with distinct properties. A nilpotent matrix is a square matrix that, when raised to some power, becomes the zero matrix. A Hermitian matrix, on the other hand, is a square matrix that is equal to its conjugate transpose.

Theorem: If a matrix is both nilpotent and Hermitian, then it must be the zero matrix.

Proof:

Let A be a matrix that is both nilpotent and Hermitian. Since A is nilpotent, there exists a positive integer k such that A^k = 0. Since A is Hermitian, A = A*.

Consider the case where k = 2. Then A^2 = 0. Since A is Hermitian, we have: A^2 = AA = AA*

= 0

Multiplying both sides by A* on the left, we get:

A*AA* = 0

Since AA* = 0, we have:

A*0 = 0

This implies that A = 0.

For the general case where k > 2, we can use a similar approach. Since A^k = 0, we have:

A^k = A^(k-1)A = 0

Since A is Hermitian, we can replace one of the A's with A*: A^(k-1)A*

= 0

Multiplying both sides by A on the left, we get:

AA^(k-1)A* = 0

Since A^k = 0, we have:

0A* = 0

This implies that A = 0.

Conclusion:

Therefore, if a matrix is both nilpotent and Hermitian, it must be the zero matrix. This theorem establishes a connection between these two classes of matrices and provides a useful tool for analyzing matrices that possess both properties

Reference:

1. Beauregard, Raymond A.; Fraleigh, John B. (1973), A First Course In Linear Algebra: with Optional Introduction to Groups, Rings, and Fields, Boston: Houghton Mifflin
2. Co., ISBN 0-395-14017-XHerstein, I. N. (1975), Topics In Algebra (2nd ed.), John Wiley & SonsNering, Evar D. (1970), Linear Algebra and Matrix Theory (2nd ed.), New York: Wiley, LCCN 76091646.
3. Archibald, Tom (2010-12-31), Gowers, Timothy; Barrow-Green, June; Leader, Imre (eds.), "VI.47 Charles Hermite", The Princeton Companion to Mathematics, Princeton University Press, p. 773, doi:10.1515/9781400830398.773a, ISBN 978-1-4008-3039-8, retrieved 2023-11-15^ Ribeiro, Alejandro. "Signal and Information
4. Processing" (PDF).^ "MULTIVARIATE NORMAL DISTRIBUTIONS" (PDF).^ Lau, Ivan. "Hermitian Spectral Theory of Mixed Graphs" (PDF).^ Liu, Jianxi; Li, Xueliang (February 2015). "Hermitian-adjacency matrices and Hermitian energies of mixed graphs". Linear Algebra and Its Applications. 466: 182–207. Doi:10.1016/j.laa.2014.10.028.^ Frankel, Theodore (2004). The Geometry of Physics: an introduction. Cambridge University Press.

   P. 652. ISBN 0-521-53927-7.^ Physics 125 Course Notes Archived 2022-03-07 at the Wayback
5. Machine at California Institute of Technology^ Trefethan, Lloyd N.; Bau, III, David (1997). Numerical linear algebra. Philadelphia, PA, USA: SIAM. P. 34. ISBN 0-89871- 361-7. OCLC 1348374386.^ Jump up to:a b c Horn, Roger A.; Johnson, Charles R. (2013). Matrix Analysis, second edition. Cambridge University Press. ISBN 9780521839402.^ Also known as the Rayleigh–Ritz ratio; name after Walther Ritz and Lord Rayleigh.^ Parlet B. N. The symmetric eigenvalue problem, SIAM, Classics in Applied Mathematics,1998