

# Theory of finite resistance corrected voltmeter or unmodified voltage collector device

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## Abstract:

In this note we have demonstrated an iterative correction technique in the measurement process of potential difference using finite resistance voltmeter. We may call it unmodified voltage collector also. We have designed and analysed the working of the device.

**Keywords:** correction resistance; iterative process.

## 1. Introduction:

Working on basics of physics is really a challenging task indeed. One such fundamental topic in physics is current electricity and instrumentation. Literature survey shows some very recent works on instrumentation due to Gingl [1,2]. In his/her work [1] Gingl designed a circuit where a voltmeter was connected in series. It was truly pointed out in the work that real voltmeters have finite input resistance, therefore, one may think that they can be used as simple resistors. In addition, voltmeters measure the voltage difference between their terminals, so it seems to be possible to calculate the current flowing through them simply using Ohm's law. It was mentioned that it may be considered as a very much innovative and thought provoking problem for the students. In the back to back work Gingl [2] explains the importance of using Arduino platform in physics education in various experiments. It was mentioned that python coded Arduino may be widely used to perform simple experiments and science projects involving various sensors and A/D converters. But it was also warned that projects and experiments should be performed with in depth understanding only and not by simple copy and paste from the web. With this background, we also decided to work on instrumentation, particularly on some other major aspect of voltmeter. We describe our study below.

Often in an electrical circuit we need to collect the voltage drop across a resistor and to use it to drive some other device or circuit. This procedure works fine only if the parallel device so connected is of infinite input impedance. Eventually it would not draw any current from the main circuit path and the said potential drop will remain unaltered. Taking a typical example of an ideal voltmeter connected across a resistor, it does not draw any current from the circuit and consequently it will measure the unmodified potential drop across the resistor.

But once we switch to the finite resistance electrical meters (reality demands so), situations will become really complicated. Parallel device with finite resistance (e.g. a real voltmeter having very large but finite resistance) will draw some current from the main circuit, hence the major circuit current will definitely drop, so also the potential difference across the resistance. This measurement problem is inherent in current electricity but it is unlike the inherent measurement problems of quantum mechanics, those we may not overcome.

Definitely this classical measurement problem of electricity can be tackled. A long and iterative correction procedure do exist to resolve the issue.

Let us now move into the problem in details with typical symbolic analysis. Let  $R_1$  is the resistance under consideration and  $I$  being the main circuit current passing through it, resulting a potential drop across  $R_1$  as  $V = IR_1$ . One may now assume that  $R_2$  is the finite resistance of the electrical measurement or voltage collector device connected across  $R_1$  in parallel. Immediately the current  $I$  will be splitted as

$$I = I_1 + I_2, \quad (1)$$

passing through  $R_1$  and  $R_2$  in respective order and the potential difference across  $R_1$  falls to

$$V' = I_1 R_1 = I_2 R_2, \quad (2)$$

in place of (we are assuming that a constant current source is connected to the main circuit)

$$V = (I_1 + I_2) R_1. \quad (3)$$

In order to regain this original voltage  $V$  one has to make some corrections in the original circuit. A simple calculation involving (2) and (3) reveals that a correction resistance of magnitude

$$R_{c1} = R_1 \frac{R_1}{R_2} \quad (4)$$

has to be connected in series with the resistance  $R_1$  to regain the voltage  $V$  across  $R_1$ . But immediately after this hardware modification further re-distribution of current occurs. Now the current through the main path and shunt device are say  $I_3$  and  $I_4$  respectively. Just before the second stage of splitting

$$V = (I_3 + I_4) \left( R_1 + R_1 \frac{R_1}{R_2} \right) \quad (5)$$

and Just after the second stage current splitting

$$V'' = I_3 \left( R_1 + R_1 \frac{R_1}{R_2} \right) = I_4 R_2 \quad (6)$$

We have illustrated few initial stages of splitting of current in figure 1.

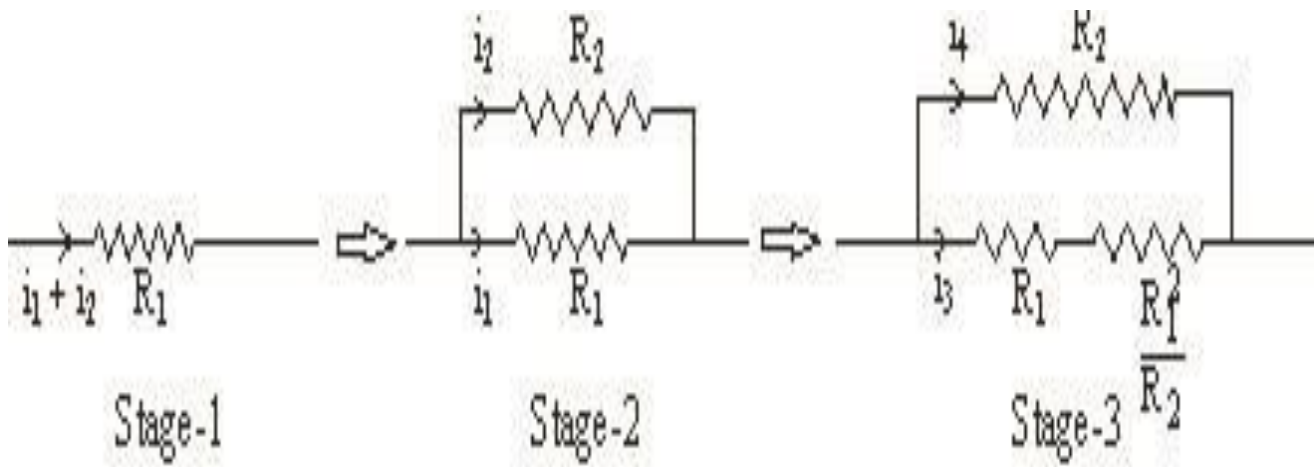


Figure 1: Illustration of splitting of current

Computation similar to previous iteration using (5) and (6) yields a stage two correction resistance

$$R_{c2} = \left[ R_1 + 2R_1 \left( \frac{R_1}{R_2} \right) + 2R_1 \left( \frac{R_1}{R_2} \right)^2 + R_1 \left( \frac{R_1}{R_2} \right)^3 \right]. \quad (7)$$

Continuing with this iterative correction technique one obtains at a stage

$$V = I_k R_1 \left[ 1 + 2 \left( \frac{R_1}{R_2} \right) + 2 \left( \frac{R_1}{R_2} \right)^2 + 2 \left( \frac{R_1}{R_2} \right)^3 + \dots \right]. \quad (8)$$

If  $R_1 > R_2$ , the major current will flow through the shunt device continuously and as time passes on, the current through the main path will become zero ( $I_k \rightarrow 0$ ) and the series sum will become infinite. On the contrary if  $R_1 < R_2$ , the series sum will tend to some finite value and one may obtain a closed form of the final iterative correction resistance from (8)

$$R_{\text{correction}} = \frac{2R_1^2}{R_2 - R_1}. \quad (9)$$

This iterative correction technique definitely works for direct current. However for alternating current, resistances have to be replaced by impedances and in addition to simple resistor, inductors and capacitors may also be present in the circuit.

Once we obtain the correction resistance, the final circuit current may be related to the constant current source  $I$  driving the circuit

$$I_k R_1 \left( \frac{R_2 - R_1}{R_2 + R_1} \right) = (I - I_k) R_2, \quad (10)$$

solving

$$I_k = \frac{iR_2(R_2 - R_1)}{R_1^2 + R_2^2}. \quad (11)$$

However it is difficult to relate  $I_k$  with the constant voltage source without the knowledge of other part of the circuit.

We already mentioned that this correction meter is nothing but a realistic voltmeter and may contain any finite internal resistance  $R_2$ . In the way this work may be considered as a major step towards improvement of voltmeter and hence instrumentation in current electricity.

**References:**

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