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A Problem Formulation & Approach of solving Stochastic Transportation Problem

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Abstract:
This paper present the study on Transportation Problem. We find out the most appropriate ways to full fill the demand of cost optimization in product manufacturing company. The mathematical formulation of the problem under consideration is optimized by using Linear Programming approach. The case study of Transportation Problem analysis shows that the minimum cost transportation the product can be archived. If the product is shipped transhipment point between manufacturing point and destination with varying objective function.

Keywords: Transportation Problem, Linear Programming

1. Introduction:

The transportation problem is a well-known stochastic application of linear programming, in which an item is to be transported from sources to destinations [1]. The transportation Problem introduced as far back as the 1940s has received wide acceptance over the years with many researchers making several improvements to suit their peculiar/present needs. Transportation is one of the key or important technical method in any organization. It may be difficult for this sector to be ignored in managerial decision making. The concept takes into account a transportation model in which any of the origin and destination can serve as an in terminate point through which goods can be temporarily received and then transhipped to other points or to the final destination [2]. The production company considers that within a given time period each shipping source has a certain capacity and each destination has certain requirements with a given cost of shipping from source to destination. The Objective Function is to minimize total transportation costs and satisfy destination requirements within source requirements [3]. Hence, the optimal planning is unavoidable or necessary so as to minimize the cost of transporting these items so as to maximize the profit for the organization. The major focus in the transhipment problem was to the first instance obtain the minimum-cost and/or minimum transportation cost, however due to technological development the minimum-durational transportation problems are now being studied. In recent times other researchers have extensively studies this Linear Programming Problems and developed different variants based on the needed objective function. There may be more than one objective to the problem, and they could be conflicting, for example, minimizing the cost of transportation as well as minimizing the shipping time. Hence, The Linear programming is introduced so that the decision maker may set formulate for the solving in a transportation problem. The supply and demand parameters can be random variables, so it becomes a stochastic transportation problem. Considers a problem formulation approach transportation problem, where the supply and demand parameters of the constraints follow extreme value distribution. Some of the cost coefficients of the objective function are a multi choice type. In an optimal solution, the number

of units to be transported should be determined while satisfying source and destination demands to ensure minimum transportation costs [4]. With the introduction of Operations Research and in particular linear programming and networking to subject areas like statistics and management, operations managers can now deal with this challenging need. Among the many linear programming problems introduced by Operations Research is the Transportation Problem. The transportation problem introduced as far back as the 1940s has received wide acceptance over the years with many researchers making several improvements to suit their peculiar/present needs. These are manufacturing company produce and service providing company. The scheduling of production has been a crucial issue for any manufacturing company because an excess or insufficient supply of productions may lead to the loss of the company both in measures of time and cost. Optimal planning and scheduling of productions will surely increase the productivity of the company. With a view to minimizing the total production cost, a manufacturing company should maintain proper scheduling of work for its labours. There is another cost associated with total cost. That is the inventory warehouse and handling cost. As a manufacturing company produces physical products, it should keep the products in stock for maintaining proper flow of shipping and for fulfilling the need of future demand. So, a manufacturing company has to spend money in keeping the inventory in stock. Here, the company should maintain an optimal level of inventory that ensures the least cost in one hand and the proper flow of inventory in another hand. Sometimes, a manufacturing company may change the labour schedule due to financial and market demand related issues. The change may be increase in labour hours spent or decrease in labour hours spent. But, this should be kept at an acceptable limit. This means, the change should be optimize with the company policy. As a result, the rescheduling of labour mix needs to be made. But, the rescheduling of labour mix should be prepared in such a way that ensures minimum labour cost and minimum production cost. A linear programming (LP) technique may be effective to schedule the labour mix in the study that ensures the minimum labour cost. In LP in the decision making process of entrepreneurs as an optimization tool for ensuring highest profit with the existence resources. Samples were taken from Energy drink or commodity organization that faced few challenges for making product. The results revealed that discontinuity needed to exist in the manufacturing of product and that concentration should be made with the production. Linear Programming considered the chance of applying these techniques for the long term manufacturing planning as the best crucial advantage. The comparative accuracy of these techniques was found as other advantage.
In this study, we will propose a new approach to the transportation problem where by the supply and demand parameters are random variables following extreme value distribution. Rather than minimizing the cost coefficient for the transportation problem, we can minimize the time for shipping, minimize the risk in shipping the items and so on. As an additional feature, each objective can have multiple aspiration levels instead of only one. Now the problem becomes a stochastic transportation problem. To overcome this difficulty, first we will use a stochastic approach to turn the probabilistic constraint into a deterministic one. Second, a general transformation consisting of binary variables is applied to select one aspiration level for each objective from multiple levels. The reduced problem then becomes a TP and it will be solved with linear programming [5].

Like all linear programming problems (LPP), the transportation problem has its objective function and constraints. The most common objective function is to schedule shipments from sources to destinations so that total production and transportation costs are minimized [6]. And the constraints are that the resources to be optimally allocated usually involve a given capacity of goods at each source and a given requirement for the goods at each destination. For the transportation problem, the source/supply points can only send out goods but cannot receive any while the sink or the demand point can only accept goods and not give out. With the diversification commodity type, size, distance to sinks etc., this transportation model becomes limited at some point. To overcome this limitation, a variant of the transportation problem with an intermediate point was introduced. This is known as the Transhipment problem. A transhipment is defined as the transfer of stock between two locations at the same level of the inventory/distribution system. The problem is to determine replenishment quantities and how much to tranship each period so as to satisfy deterministic dynamic demand at each location at minimal cost. The planning horizon is finite and no back orders are allowed [7]. It canal so be seen as transportation of goods/containers to an intermediate destination, before it is shipped to another destination. There may be several reasons for such a change. One of which is to trans-load i.e. changing the means of transport during the journey. Other reasons could be to combine small shipments into a large shipment, dividing the large shipment at the other end. Whatever the reason, this model takes into consideration a multi-phase transport system where the flow of material-raw and/or finished goods and services are taken through the intermediate point which is between the origin and the destination. The whole stock is expected to pass through these points of reloading before the goods are finally sent to their destination [8]. Though the transhipment problem is an extension or improvement to the transportation problem its optimum solution is found by easily converting the transhipment problem into an equivalent transportation problem and solving using the usual transportation techniques. The availability of such a conversion procedure significantly broadens the applicability of the algorithm for solving transportation problems. The conventional Transportation Problem can be represented as a mathematical structure which comprises an Objective Function subject to certain Constraints. The major focus in the transhipment problem was to initially obtain the minimum-cost and/or shortest transformational route, however due to technological development the minimum-durational transportation problems are now being studied. In recent times other researchers have extensively studies this Linear Programming Problems and developed different variants based on the needed objective function.

1.1 Components of a Linear Programming Approach:

Like many other kinds of optimization approaches, LP is a mathematical model which has different components. The most important components of a LP model are:

- Defining key - decision variables
- Setting objective functions
- Writing mathematical expressions for constraints
- Non-negativity restriction
- Solving the mathematical model

1.2 Mathematical Formulation of the Transportation Programming:-

The Transportation Problem is considered

If k_{ij} represents the product transported from the source to the destination, than the transportation model can be defined as follows:

Find k_{ij}, $i= 1,2\ldots.p$

$j= 1,2\ldots.q$

$min\ Z= \sum_{i,j} e_{ij}\, k_{ij}$,

Subject to

$$\sum_{j=1} k_{ij} \leq c_i, \forall_i$$

$$\sum_{i=1}^{p} k_{ij} \geq d_j, \forall_i$$

$$\sum_i c_i = \sum_j d_j$$
$$k_{ij} \geq 0, \forall_{i,j}$$

Where e_{ij} the transportation cost per unit is $k_{i,j}$ is the amount shipped, c_i is the amount shipped,c_i is the amount of supply at source i and d_j is the amount of demand at destination.

Now, we consider a mathematical technic for a stochastic multi dimension transportation problem with an extreme value distribution as follows [9].

Let min

$\{q_i, z_i\}$

s.t. $f_i(x) + q_i - z_i = h_1, h_2 \dots \dots \dots h_r, r = 1,2 \dots \dots . . l$

$z\ (\sum_{j=1}^{q} k_{ij} \leq c_i) \geq 1 - \gamma_i, i = 1,2 \dots \dots \dots p; 0 \leq \gamma \leq 1$

$z(\sum_{i=1}^{p} k_{ij} \geq d_j) \geq 1 - \delta_j, j = 1,2 \dots \dots . q; 0 \leq \delta_j \leq 1,$

$$k_{ij} \geq 0, q_i p \geq 0,$$
$$\sum_{i=p}^{p} c_i = \sum_{j=1}^{q} d_j$$

Where $f_i(x)$ is the linear function of the i_{th}, h_i is the aspiration of the i^{th},

x_{ij} is the product shipped, c_i is the product of supply at source i, d_j is the product of demand at destination j, q_i is the negative deviation variable and p_i is the positive deviational variable.

Lemma: - 1.

Constraint into a deterministic constraint using the disjoint chance-constrained method.

The supply and demand constraints were considered:

i) Only $c_i = i = 1,2 \dots \dots \dots p$ follows extreme value distribution

ii) Only $d_j = j = 1,2 \dots \dots \dots q$ follows extreme value distribution

iii) Both $c_i = i = 1,2 \dots \dots \dots p$ *and* d_j, j= 1,2…….q follow extreme value distribution

This three different models transformed constraint is then considered here as the probabilistic constraint into a deterministic linear constraint:

$$\sum_{j=1}^{q} k_{ij} \leq \alpha_i - \beta_i[1n\{-1n(\gamma_i)\}]$$

The Probabilistic constraint transformed in to a deterministic linear constraint:

$$\sum_{i-1}^{p} x_{ij} \geq \alpha_j - \beta_j[1n\{1n(1 - \delta_j)\}]$$

Now, a deterministic linear programming transportation problem with an extreme value distributions model will be obtained.

Lemma: - 2.

Let min $\{q_i, z\}$

s.t. $f_i\ (k) + q - z_i = h_1, h_2 \dots \dots \dots h_r, r = 1,2 \dots \dots \dots l$

$$\sum_{j=1}^{q} k_{ij} \leq \alpha_i - \beta_i[1n\{-1n(\gamma_i)\}], i = 1,2 \dots \dots \dots p$$

$$\sum_{i=1}^{p} k_{ij} \geq \alpha_i - \beta_j\left[1n\left[-1n\left(1-\delta_j\right)\right\}\right], j = 1,2 \ldots \ldots \ldots$$

$$k_{ij} \geq 0, q_i, z_i \geq 0$$

$0 \leq \delta_j \leq 1$

$0 \leq \gamma_i \leq 1,$

Where $\sum_{i=1}^{m} \alpha_1 - \beta$ [1n {-1n (γ_i)}] $\geq$ $\sum_{j=1}^{n} \alpha_j - \beta_j[1n\{-1n(1-\delta_j)\}]$

Is the feasibility condition.

2. Case Study:

Example I.

The Energy drinks are in very demand in summer season at each four destination distribution centres. The transportation time cost is an essential factor in a transportation planning programme as well as the transportation cost. The manufacturing time at production centres depends on the current supply, machine condition, skilled labour etc. Transportation time is related to distribution of a product in due time to destination centres. The transportation time cost t_{ij} and cost coefficient c_{ij} from each source to each source to each destination are considered in table 1.

The energy drinks supply company is seeking to minimize the transportation time and minimize the cost of transportation. The initial target values are 112000 or 113000 hours and ₹150000 or ₹160000 respectively.

The transportation problem approach has been considered, in which the supply and demand parameters follows extreme value distribution.

Transportation time t_{ij} and the cost coefficient c_{ij} from each source to each destination.

Let

No	Route k_{ij}	Transportation time t_{ij}	Cost coefficient c_{ij} (₹)
1	(1,1):k_{11}	12	21
2	(1,2):k_{12}	15	25
3	(1,3):k_{13}	19	30
4	(1,4):k_{14}	24	34
5	(2,1):k_{21}	16	27
6	(2,2):k_{22}	18	28
7	(2,3):k_{23}	9	15
8	(2,4):k_{24}	17	26
9	(3,1):k_{31}	24	34
10	(3,2):k_{32}	12	24
11	(3,3):k_{33}	25	37
12	(3,4):k_{34}	28	40

min $\{z_1, z_2\}$

s.t. $\sum_{i=1}^{3} \sum_{i=1}^{4} k_{ij} + q_1 - z_1 = 1112000w_1 + 113000(1-w_1)$

$$\sum_{i=1}^{3}\sum_{j=1}^{4} k_{ij} + q_1 - z_1 = 150000w_1 + 160000(1 - w_1)$$

$$\sum_{j=1}^{4} k_{1j} \leq 2994.502$$

$$\sum_{j=1}^{4} k_{2j} \leq 2495.908$$

$$\sum_{j=1}^{4} k_{3j} \leq 1996.9989$$

$$\sum_{i=1}^{3} k_{i1} \geq 1707.038$$

$$\sum_{i=1}^{3} k_{i2} \geq 1505.94$$

$$\sum_{i=1}^{3} k_{i3} \geq 1254.452$$

$$\sum_{i=1}^{3} k_{i4} \geq 1003.147$$

$$k_{ij}, q_u, f_u \geq 0,$$

$$i = 1,2,3 \quad j = 1,2,3,4 \quad u = 1,2.$$

$$w_l = 0 \ \ or\ 1, \qquad l = 1,2.$$

Checking that the feasibility condition is satisfied:

$$\sum_{i=1}^{p} \alpha_i - \beta_i[1n\{-1n(-\gamma_i)\}] = 7487.399 \geq \sum_{j=1}^{n} \alpha_j - \beta_j[1n\{-1n(1-\delta)\}] = 5470.577$$

$$k_{12} = 736.904$$

$$k_{13} = 1254.452$$

$$k_{14} = 1003.147$$

$$k_{22} = 1707.038$$

$$k_{22} = 769.037$$

$$z_1 = 0$$

$$z_2 = 0$$

$$q_1 = 12112.028$$

$$q_2 = 2213.805$$

$$where\ w_1 = 0\ and\ w_{2=}0$$

The outcome result that the decision variable are zero 113000 hours and zero positive deviation which means that the transportation time achieved and the minimize the cost of transportation has ₹160000 and zero positive deviation, which means that transportation cost also reached the desired level exactly.

Example II

In the manufacturing hub, commodity K from a number of factories where they are produced in a factories m to several warehouses in sub-urban area locate at the centre, their n warehouse the total supply of goods to be transported out from each factory and the total number of goods demanded by warehouse location is q. Let the cost of distribution these goods between the various factories and warehouse be r.

The Transportation Problem defined as:

p_i denotes the total of goods from factory i,
$where\ i = 1,2 \ldots \ldots \ldots m$

a_j denote the total demand on goods at warehouse j,
$where\ j = 1,2 \ldots \ldots \ldots n$

r_{ij} denote the unit transportation cost from factory i to warehouse j
t_{ij}denotes the quantity of goods distributed from factory i to warehouse j

$if\ z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} k_{ij}$

Then our objective function is:
$min\ z \sum_{i=1}^{m} \sum_{j=1}^{n}\ c_{ij} k_{ij}$
Then our objective function is:
$min\ z = \sum_{i=1}^{m}\ \sum_{j=1}^{n}\ c_{ij} k_{ij}$
Subject to the following constraints:

$$\sum_{j=1}^{n} k_{ij} \leq p_i\ (Supply\ constraint)$$

$$\sum_{i=1}^{m} k_{ij} \geq q_i\ \ (Demand\ constrant)$$

$k_{ij} \geq 0$ *(Non-negative)*

$$\sum_{i=1}^{m} p_i = \sum_{j=1}^{n} q_j\ \ (Balance\ constraint)$$

The mathematical model representation of the transportation problem:

$min\ z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}k_{ij}$

Subject to

$$\sum_{j=1}^{n} k_{ij} \leq s_i\ (\forall_i, i = 1,2 \ldots \ldots \ldots m)\ \ldots \ldots \ldots \ldots \ldots \ldots (i)$$

$$\sum_{i=1}^{m} k_{ij} \geq d_j\left(\forall_j, j = 1,2 \ldots \ldots \ldots n\right) \ldots \ldots \ldots \ldots (ii)$$

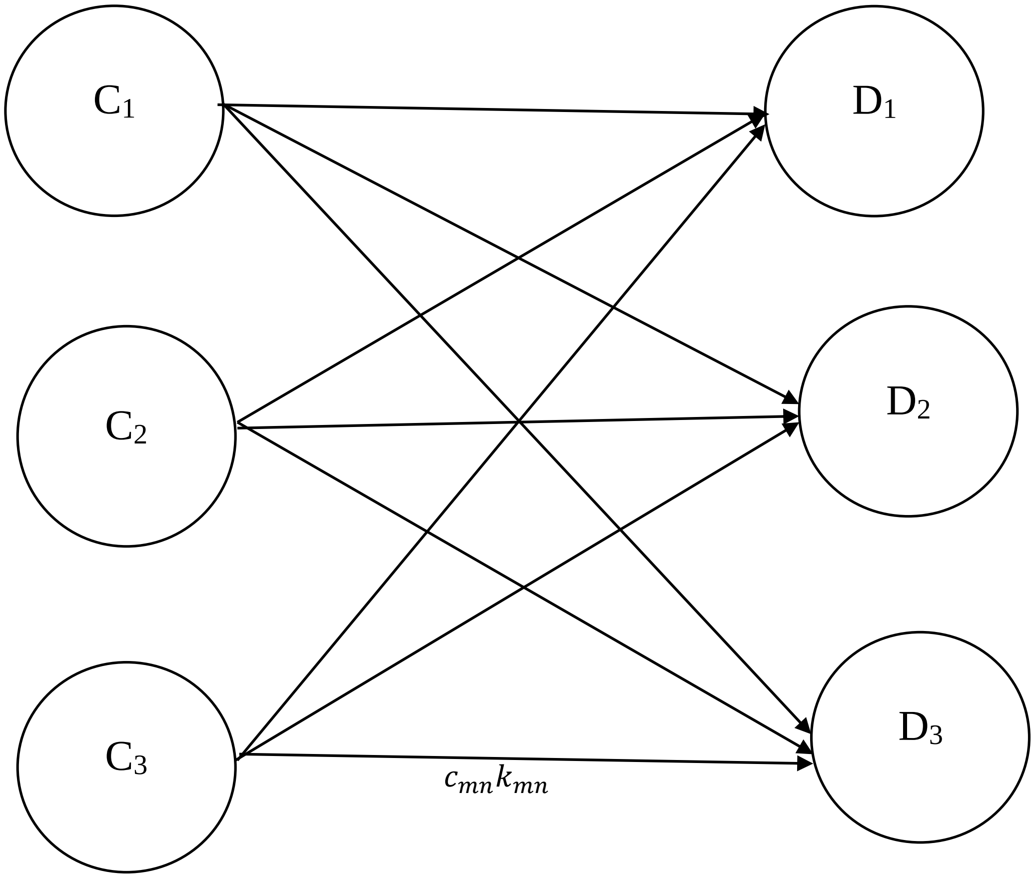
$k_{ij} \geq 0$............ *(iii)*

The constraints *(i)* and *(ii)* express as an inequality condition, the several demand centres in transportation problem.

The Transportation Problem is here represent in a table:

Sources (i)	***D₁***	***D₂***	**……..**	***Dₙ***	***Supply (aᵢ)***
S₁	$c_{11}k_{11}$	$c_{12}k_{12}$	………	$c_{1n}k_{1n}$	$p_1A = \pi r^2$
S₂	$c_{21}k_{21}$	$c_{22}k_{22}$	………	$c_{2n}k_{2n}$	$p_2A = \pi r^2$
…			………		
Sₘ	$c_{m1}k_{m1}$	$c_{m2}k_{m2}$	……….	$c_{mn}k_{mn}$	p_m
Demand (qⱼ)	q_1	q_2	……..	q_n	$\sum_{i=1}^{m} p_i = \sum_{j=1}^{n} q$

The Transportation Problem can also be presented as *m* supply and *n* demand.



Balance and Unbalance Transportation Problem:
The Transportation Problem in reality most often does not appear balanced.
i.e. the total number of goods demand does not always equal the total number of goods supplied

$$\sum_{i=1}^{m} p_i \neq \sum_{j=1}^{n} q_j$$

Where equation (iii) is unbounded transportation problem.
$\sum_{i=1}^{m} p_i > \sum_{j=1}^{n} q$: This case is where we have the total supply exceeding the total demand.
$\sum_{i=1}^{m} p_i < \sum_{j=1}^{n} q$: this second case of the unbalance type of the transportation problem is this case where total supply is exceeded by total demand.

3. Summary:

Analysis the work from which our data is sourced, it was observed that the optimal solution obtained in the work was obtained. In this paper the optimal solution was arrived at graphically and the result gives an optimal solution.

4. Conclusion:

In this paper, we have explored a study of transportation problem when the supply and demand parameters are the stochastic type and follow extreme value distribution.

The data of a footwear manufacturing company were analysed to make a schedule of production for next two months. The mathematical formulation was carried out by LP technique and the problem was solved. The transportation problem remains a major problem in distribution firms, and as such should be effectively given managerial concern. Optimization of transportation cost is necessary as to meet profit archived.

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