

Nilpotent matrix and Singular matrix

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Lecture

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Abstract: The nilpotent matrix and singular matrix same value satisfying by 0.

1. Introduction:

The nilpotent matrix and singular matrix same value Satisfying by 0.

$$AB=BA=I=A^T=0.$$

Nilopotent matrix:

A nilpotent matrix is a square matrix A for which there exists a positive integer k such that $A^k = 0$ where 0 is the zero matrix of the same size as A

The smallest such positive integer k is called the index or degree of nilpotency of the matrix A

Example: $A^k = A^T = \begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix}$

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$$= -4 + 4 = 0$$

Properties of Nilpotent Matrices:

- * Eigenvalues: All eigenvalues of a nilpotent matrix are zero.
- * Determinant: The determinant of a nilpotent matrix is always zero.

Singular matrix:

A singular matrix is a square matrix whose determinant is equal to zero. In simpler terms, if you calculate the determinant of a square matrix and the result is 0, then that matrix is classified as a singular matrix.

Here are some examples of singular matrices:

Example 1 (2x2 matrix):

$$A = \begin{pmatrix} 2 & 4 \\ 1 & 2 \end{pmatrix}$$

The determinant of A is $(2 \times 2) - (4 \times 1) = 4 - 4 = 0$. Therefore, matrix A is singular. Notice that the second row is a scalar multiple (2 times) of the first row, indicating linear dependence.

2. Conclusion:

The nilpotent matrix and singular matrix same value Satisfying by $AB=BA=I=0$. Is called by “Equal matrix”

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