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# Quantum Information Measurements in the Ion Trap

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**Abstract**

In this paper, we look into quantum information measures in ion traps using the harmonic oscillator model to describe trapped potentials and ion dynamics. Our findings show that, in the steady-state situation, mutual information and synchronisation measures behave similarly. In addition, we investigate how these metrics vary in the quench model and how they are affected by coupling and external noise, such as an external magnetic field. Moreover, for a proposed target ground state, we determine the circuit depth and analyze the effects of the external magnetic field and coupling constant, highlighting their dynamic evolution over time. We also discuss the coherent state of a single ion in a trap, noting an inverse relationship between complexity and fidelity-where increased fidelity corresponds to decreased system complexity, indicating a more ordered state with improved control and optimization. However, at higher system frequencies, complexity increases due to intricate interactions and rapid state changes, necessitating advanced control mechanisms.

**Keyword:** Ion Trap, synchronization, quantum information, Steady-State Approximation**.**

## 1. Introduction.

The developing quantum algorithms that outperform traditional approaches, such as big integer factorisation, has made quantum computing a primary emphasis in current physics. Among the several techniques to physically manufacturing quantum computers, the ion trap method presented by Cirac and Zoller [1] appears to be particularly promising. Extensive tests have confirmed the usefulness of ion traps as a tool for realistic quantum computing. An ion trap is a device that employs electric and/or magnetic fields to confine charged particles (ions) in a defined area of space. This confinement allows for ion manipulation and analysis; in fact, the capacity to precisely manipulate individual ions enables accurate quantum processes, while trapped ions' lengthy coherence durations assure stability throughout complicated computations [2]. The scalability of ion trap systems allows for the building of bigger quantum systems, which are supported by high-fidelity quantum gates that reduce operational faults. Furthermore, ion traps aid in the creation of entangled states, which are required for quantum communication and distributed computing. In this context, the potential in ion traps is frequently approximated by a harmonic oscillator, giving a well-established framework for analysing the motion and interactions. The interactions between ions within a trap, including those in optical or electromagnetic resonators, can be modeled as coupled harmonic oscillators, which are vital for controlling quantum states and performing quantum operations such as entanglement. These interactions can enter various coupling regimes—weak, strong, and ultra-strong—each of which plays a critical role in enhancing the performance and scalability of quantum computers [4,5]. In the realm of quantum computing, particularly within the framework of Hamiltonian dynamics of trapped ion systems, a nuanced understanding of various quantum metrics is essential. For example, entanglement entropy measures the quantum correlations between subsystems, indicating how much information is shared. This is important for quantum algorithms and protocols, such as error correction and cryptography. An other metric is the computational complexity which assesses the resources needed for quantum computations, including the number of qubits and the depth of the quantum circuit. This reflects the difficulty of quantum operations and the efficiency of algorithms. High entanglement entropy often leads to increased computational complexity because maintaining entanglement requires more complex and deeper circuits. On the other hand, by arranging quantum gates in sequences, efficient quantum algorithms are formed, enabling quantum computers to solve problems beyond the capabilities of classical computers1. The study of interaction between quantum gates and the wave function is important; transforming a reference state $|\psi R\rangle$ into a target state $|\psi T\rangle$ involves applying a unitary transformation U, achieved through sequences of universal gates. Optimizing these gate sequences is crucial due to the infinite possible paths to the same target state. The purpose of this research is to further the understanding of quantum metrics, especially fidelity, synchronisation, and mutual information, in the context of ion traps using the harmonic oscillator approximation. Particular emphasis is placed on the impact of coupling and external fields on these metrics. We validate previous findings that mutual information and synchronisation measures operate similarly in the steady state. External noise (such as an external magnetic field) has an influence on these measurements in two specific models: one involving two linked Vander Pol oscillators and the other using two qubits in optical cavities with driving forces attached. External noise might disrupt the system and change its behaviour. By researching these factors, we want to understand how synchronisation and mutual information measures evolve and adapt to various. The goal of this study is to improve our knowledge of quantum metrics, including fidelity, synchronisation, and mutual information, in the setting of ion traps using the harmonic oscillator approximation. These measurements are heavily influenced by coupling and external fields. We corroborate prior findings that mutual information and synchronisation measures behave similarly in the steady state. External noise influences these observations in two models: one with two connected Vander Pol oscillators and another with two qubits in optical cavities with driving forces attached. External noise may interrupt the system and alter its functioning. By examining these characteristics, we want to understand how synchronisation and mutual information measures evolve and adapt to varied

## Mathematical Foundations by Harmonic Oscillator Model:

A device with a powerful radio-frequency electric field restricts an ion's mobility in the y and z directions, confining it to the x direction. In the x direction, the ion is poorly restricted by an electrostatic field that is approaching its minimum. A harmonic oscillator can approximate the axial potential of the system. Ion transport involves adjusting the electrostatic field intensity to shift the potential well. With careful tuning, the harmonic well strength remains constant throughout the operation. The system is modelled as a trapped ion vibrating in a harmonic well along the axial x-direction at frequency ω [21-23]. The Hamiltonian describes the dynamics of a system using the equation Hˆ0 = aˆ and ˆa † are raising and lowering operator. By moving the harmonic potential well along the trap axis while maintaining a constant curvature, the Hamiltonian that describes the system becomes time-dependent and is expressed as

Hˆ (t) = pˆ 2 2m + 1 2 mω2 [ˆx − d(t)]2

If we assume the initial state of ion is the motional ground state of the harmonic oscillator centered at x = 0, i.e. |Ψ(0)⟩ = |0⟩, the motional state of the ion remains in coherent state throughout the process, i.e. |Ψ(t)⟩ = |α(t)⟩, with the amplitude α(t) given by the formula [24]

α(t) ≡ r mω 2ℏ d(t) − e −iωt Z t 0 ˙d(t1)e iωt1 dt1 where, the coherent state amplitude follows from Equation are given by

α1(t) = r mω 2ℏ d0(1 − e −iωt),

α2(t) = r mω 2ℏ L sin2 πt 2T − πL πe−itω + iT ω sin πt T − π cos πt T 2 (π 2 − T 2ω2 ). Although the potential well stops its movement at time t = T, the ion remains in oscillation For two coupled ions, the dynamics and behavior of the system become more complex. In principle, the primary interaction between the ions is supposed to be the Coulomb force which couples the motion of the ions. The ions are confined within the same trapping potential, which means changes in the potential affect both ions. By varying the frequencies of the trapping potential, one can control the oscillatory motion of each ion. This can be achieved through adjustments in the RF or DC fields. As the trap frequencies change, the ions' oscillation frequencies and coupling dynamics also change and this can lead to interesting phenomena such as mode splitting and energy transfer between the ions. Let us consider the following Hamiltonian

**H = ω1aˆ † aˆ + ω2 ˆb †ˆb + g ′ (ˆa + ˆa † )(ˆb + ˆb † )**

where this form of Hamiltonian is frequently used in the study of quantum interactions in ion traps and cavity QED, where different modes (either motional or electromagnetic) are coupled.2 The interaction term g ′ (ˆa + ˆa † )(ˆb + ˆb † ) allows for energy exchange between the two modes, enabling processes like quantum state transfer, entanglement generation, or quantum gates in ion trap systems [2]. One may examine a system characterized by Hamiltonian (2.10) with a pair of harmonic oscillators engage in an interaction of significant intensity, denoted as the "position-position" coupling. By imposing an external magnetic field with the symmetric gauge $A^{\rightarrow} = B^2 (x^2 - x1)$, therefor, the Hamiltonian can be written as We investigate how to use the above results to discuss different measures related to the quantification of information, this may help to understand the effect of magnetic field on the dynamics of system. It should be mentioned that the wave function is the general Gaussian form and scaling and entangling operators preserve this form of the wave function where we begin and end with a Gaussian wave function.

**Quench and Steady-State Approximation.**

The quench model involves suddenly changing a system parameters, like interaction strength, to study non-equilibrium dynamics which is crucial for understanding quantum phase transitions and entanglement growth, providing insights into complex quantum behaviors that are difficult to simulate classically [30]. Here, we employ a realistic quenched model, where at t = 0, the frequencies ω1, ω2, and the coupling parameter g are suddenly quenched from constant initial values to new constant final values:

ωj (t) = ( ωij t = 0 ωf j t > 0 , g(t) = ( 0 t = 0 g t > 0 (2.21)

where j = 1, 2. The solutions of the Ermakov equations now take the forms h 2 1 (t) = Ω 2 f1 − Ω 2 i1 2Ω2 f1 cos (2Ωf1 t) + Ω 2 f1 + Ω2 i1 2Ω2 f1 h 2 2 (t) = Ω 2 f2 − Ω 2 i2 2Ω2 f2 cos (2Ωf2 t) + Ω 2 f2 + Ω2 i2 2Ω2 f2.

The problem is simplified to the ground state of two coupled harmonic oscillators. Before ending this section, let us mention the steady-state approximation that will be employed in the subsequent section. Under this approximation, the wave function is given by (2.17) in which one gets.

**The quantum measurements for Hamiltonian behavior.**

Consider a system that begins in a specified initial state and evolves over time under a specific Hamiltonian, such as (2.11), while measuring various time-dependent quantities to study metrics such as circuit depth and synchronicity. As mentioned, the depth of a quantum circuit, defined as the number of sequential layers of quantum gates, directly influences execution time and error susceptibility. Thus, optimizing circuit depth is essential for efficient quantum computations. Meanwhile, synchronicity in quantum circuits ensures that gate operations are well-aligned and coordinated, thereby minimizing errors and maximizing coherence by mitigating timing-related issues and decoherence. In principle, to transform a reference state |ψR⟩ into a target state |ψT ⟩, a unitary transformation U is applied as follows: this transformation is achieved through a sequence of universal gates. Finding the optimal sequence of gates, known as optimizing the trajectories, is essential due to the infinite possible sequences that can produce the same target state. The depth of a quantum circuit, which refers to the number of sequential gate layers, is closely tied to its computational complexity, in a way that a deeper circuit can perform more complex computations by introducing more entanglement and intricate manipulations of quantum states. However, this also increases susceptibility to errors and decoherence, affecting the computation's fidelity. In this context, the depth of circuit might be a metric in the context of quantum information. The aim of this section is to examine a quantum system model to measure the quantum circuit depth required to generate the output state from the system's ground state. The reference state is a factorized Gaussian state as. These gates/operators are pivotal in constructing the circuit, with Gaussian wave functions serving as target states. By applying scaling and entangling gates appropriately, the reference frequency ωR is adjusted to the target frequency. In the quenched model for a target state given by (2.17), we find the depth of circuit as

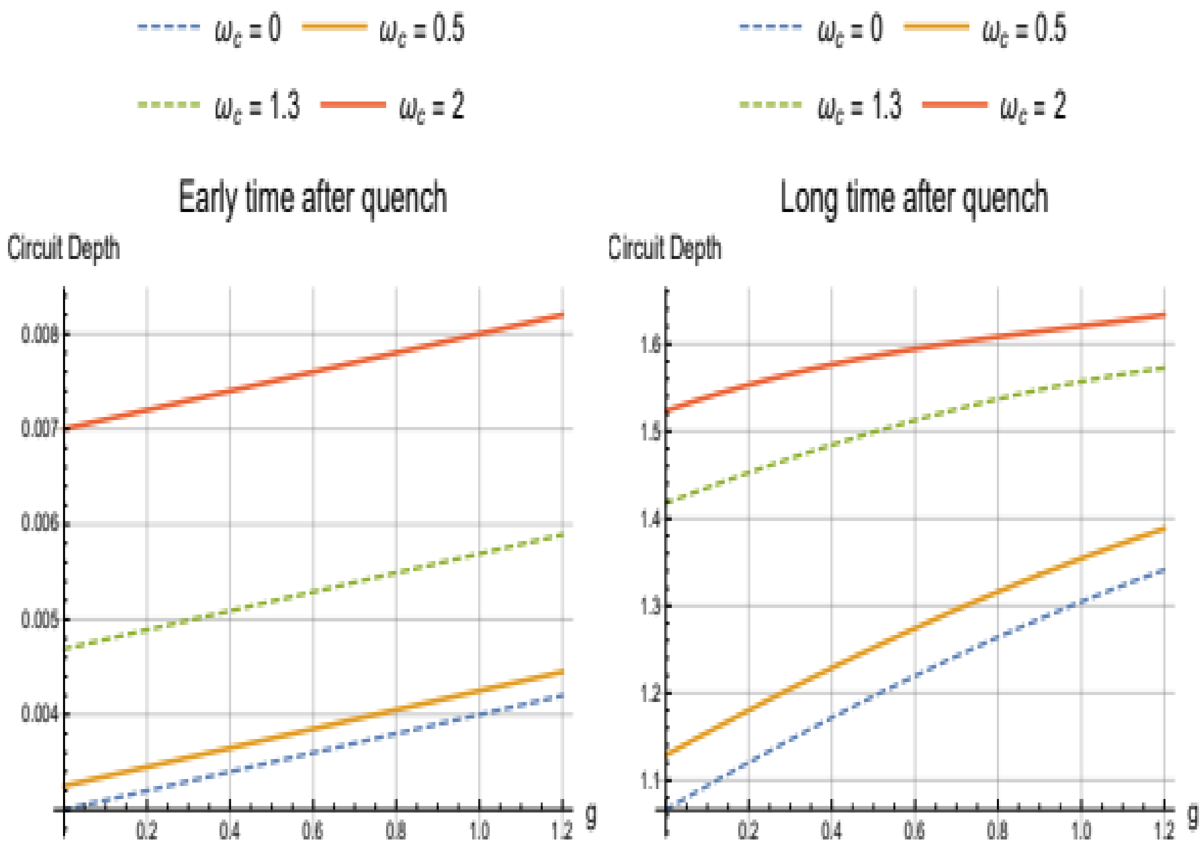


Fig.no.2 where A's are given by (2.20) in which we used (2.22). Making use of the steady-state approximation with equation (2.23), the formula (3.2) matches the results of a toy model from [31]. In Figure 1, we have depicted the circuit depth for various values of the external magnetic field and coupling constant. As illustrated, an increase in the coupling constant results in a corresponding increase in the circuit depth. Additionally, during the initial period following the quench, the circuit depth is observed to increase with rising magnetic field strength. As shown in the right panel of Figure 1, this pattern remains consistent over time. In Figure 2, we have illustrated the circuit depth for various external magnetic field strengths. At the early time after quench we observe the linear growth of circuit depth in a way that the external magnetic field ceases to have a uniformly increasing effect on circuit depth.

Synchronization is a fascinating phenomenon where two or more systems coordinate their behavior over time. This concept appears in various forms, from classical mechanics to quantum mechanics, and it plays an important role in understanding complex systems [32]. Classical synchronization happens when systems like coupled pendulums or fireflies flashing together align their behaviors over time. This means that their actions become coordinated and follow similar patterns as time goes on. However, in quantum mechanics, synchronization takes on a different way, in fact, quantum synchronization examines how quantum systems, such as ions in traps or coupled oscillators, synchronize their states or observables over time. This involves studying quantum correlation measures like fidelity, mutual information, and other entanglement-related metrics. One interesting thing about quantum synchronization is its dependence on non-classical effects and interactions.

**The synchronization and Reciprocal Data.**

Synchronisation is a fascinating process in which two or more systems coordinate their behaviour across time. This idea, present in both classical and quantum physics, is crucial for comprehending complicated systems [32]. Classical synchronisation occurs when systems, such as connected pendulums or fireflies, align their behaviour across time. Over time, their acts grow more coordinated and predictable. In quantum mechanics, synchronisation refers to how quantum systems, such trapped ions or linked oscillators, synchronise their states or observables throughout time. This includes investigating quantum correlation measurements such as fidelity, mutual information, and other entanglement-related metrics. One interesting thing about quantum synchronization is its dependence on non-classical effects and interactions. For example, the presence of coupling between quantum systems or external.

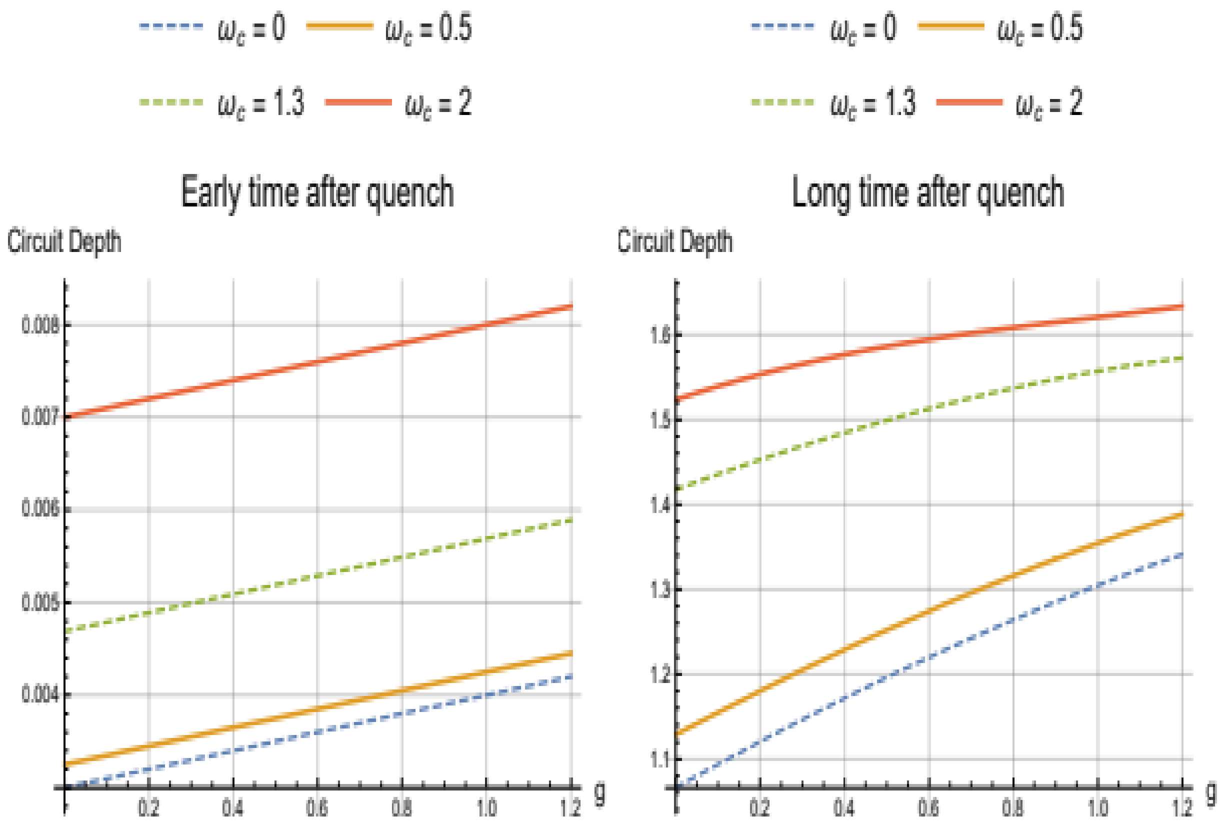


Figure 2: Schematic diagram of depth of circuit as a function of ωRt for early time after quench (left panel) where we set t = 10−3 and for different values of magnetic field. In the right panel we consider long time after the quench. In both we set g = 1 and ωf2 ≈ ωf1 = 2. fields can significantly influence synchronization behavior. Studies in this field examine how these interactions influence the stability and behavior of quantum systems. Classical synchronization is related to the Pearson correlation coefficient which is used as a measure for quantifying the temporal correlation between two classical trajectories. For two variables say as A and B, it is defined by (see Ref. [33] and Ref.s therein)

$$CA,B = AB - A\ B / A2 - A\ 2\ (B2 - B\ 2\ )^{1/2},$$

where overline stands for the average value. Pearson measure is usually used to quantify the linear dependence between two variables, ranging from +1 (total positive correlation) to -1 (total negative correlation), with 0 indicating no correlation. This coefficient can also be used in quantum synchronization by analyzing the time-dependent expectation values of quantum operators, helping to capture synchronization in quantum systems, like coupled harmonic oscillators. On the other hand, Ref. [34] introduced the following measure to quantify the level of synchronization of coupled continuous variable quantum systems. this can be used as a measure to quantify the degree of synchronization between two coupled harmonic oscillators by evaluating the deviations in their positions (x1, x2) and momenta (p1, p2). A higher value indicates stronger synchronization, implying closer alignment of the oscillators' motions and a low synchronization can be due to large mean values or variances of x1 − x2 and p1 − p2. In quantum systems, besides synchronization, mutual information is also important. It measures the total shared information between subsystems and shows their correlation and entan9 glement. Mutual information helps us understand how information flows and how subsystems are connected and it is defined by

$I = S(\rho A) + S(\rho B) - S(\rho AB)$..

where $S(\rho) = -Tr(\rho \log \rho)$ is the Von Neumann entropy. In Ref. [20], the authors demonstrated that for two models, namely, two coupled Van der Pol oscillators and two qubits inside driven optical cavities, the mutual information and synchronization measure exhibited similar behavior in the steady-state case. As a result, they concluded that mutual information could be used as a synchronization measure. In Figures 3 and 4, synchronization and mutual information are plotted with respect to the difference in natural frequencies of the two coupled harmonic oscillators: ω2 − ω1. These plots are generated using the ground state wave function with parameters specified in (2.23). We observed that in this case, their behaviors were consistent with the findings reported in Ref. [20]. Additionally, we found that a higher external field value reduced both synchronization and mutual information. We also observed that increasing the coupling constant led to higher mutual information and lower synchronization. Thus, while a higher coupling constant results in higher mutual information, it does not necessarily lead to more coordinated behavior. This phenomenon suggests that stronger connections may lead to more shared information, but also to less synchronized actions.

## One Ion in a Harmonic Potential Well.

The behaviour of a particle vibrating in a harmonic well may be investigated using quantum information metrics such as fidelity and complexity. These metrics assess the accuracy of measuring particle states and the complexity of characterising their behaviour. In state evolution, fidelity quantifies the distance between close states, whereas state complexity relates to the difficulty of expressing or controlling the state.The fidelity at time t is defined as [35]

$$F(\alpha(t)) = \exp -|\alpha(t) - \alpha(0)|\ 2$$

Time evolution of synchronization for the quench model under various conditions. For the top plots, the coupling constant is fixed at g = 1 and the frequency ωc is varied between 1 (blue) and 3 (red). The bottom plots maintain a fixed frequency ωc = 1, while the coupling constant is set to g = 0.5 (blue) and g

= 1.5 (red). The right plots in both rows also highlight the average synchronization values over time. Mutual information evolution for the quench model with different parameters. Top row: Coupling constant set at g = 1, with frequencies ωc = 1 (blue) and ωc = 3 (red). Bottom row: Frequency fixed at ωc = 1, with coupling constants g = 0.5 (blue) and g = 1.5 (red). Additionally, the right plots in both rows illustrate the average mutual information values over time for all scenarios, This inverse relationship can be understood by considering that higher fidelity often entails more precise control or optimization of the system. When we have better control over a system, it becomes easier to manage and avoids unexpected changes. This makes the system smoother and more efficient, reducing its complexity. So, higher precision leads to a more orderly and less complicated system. On the other hand in Fig. 8, we observe that at higher frequencies, the system exhibits increased complexity. As frequency increases, the system becomes more complex because its parts must interact more quickly and precisely. This leads to greater interdependence and coordination among the components. Higher frequencies require the system to adapt to faster changes and more frequent adjustments, making it more complicated overall. This aligns with theoretical expectations, as high-frequency operations need more advanced control and coordination. To summarise, complexity may be calculated using many theoretical approaches, including the Nielsen geometric method, Margolus Levitin, and Lloyd methods [41-43]. Previous research have studied how an electric field affects the pace of complexity. References [39, 44, 45] investigated the complexity rate in harmonic oscillator systems with an electric field. These investigations show that frequency is a key factor in determining the complexity rate. Additionally, the presence of an external electric field lowers the upper limit of the complexity rate, emphasising the importance of both frequency and external fields in influencing complexity dynamics.

## 2. Conclusion

This work explores synchronisation and mutual information in coupled harmonic oscillators under external magnetic fields and different coupling constants. First, we evaluated both in steady-state with the difference in natural frequencies of the two connected harmonic oscillators: $\omega 2 - \omega 1$. Our findings show that increasing the external magnetic field reduces synchronisation and mutual information. A greater coupling constant increases mutual information but decreases synchronisation. Stronger connections between oscillators may not necessarily result in better coordinated behaviour, despite more information sharing. Instead, greater contacts might result in less synchronised behaviours, indicating a complicated relationship between these two characteristics. Next, we investigated synchronisation and mutual information in a quench model. Stronger magnetic fields lead to less synchronisation, indicating less coordination among oscillators. Increased magnetic fields may affect oscillator synchronisation. However, reciprocal information behaves differently. The association begins at zero and gradually increases over time. The average value increases with both magnetic field intensity and coupling constant, suggesting that greater interactions result in more exchanged information. However, increased mutual information does not always imply improved synchronisation. These findings emphasise the requirement for synchronisation. Overall, our findings highlight the various ways in which synchronisation and mutual information adapt to changes in external factors. The observed behaviours provide vital insights into how synchronisation and information exchange work in linked systems. It was noticed. This study explored how circuit depth in two linked harmonic oscillators changes with coupling constants, external magnetic fields, and reference frequency (ωR). Initially, larger coupling constants and stronger magnetic fields improve circuit depth.

As coupling constants grow, high magnetic fields reduce circuit depth, causing the pattern to shift over time. Initially, larger ωR lowers circuit depth.

We analysed the faithfulness and complexity of a single ion in a harmonic potential well. The study found that increased fidelity reduces system complexity and improves control, leading to smoother and more efficient performance. Higher frequencies require faster and more accurate interactions between system components, resulting in increasing complexity. These findings emphasise the delicate balance between fidelity and frequency in defining the system's complexity.

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