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Quinary Transcendental Equation Involving Palindrome number

$$\sqrt[3]{p^2-q^2}+2\sqrt{r^2-2s^2}=121t^2$$

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Abstract:
In this study, we propose a novel methodological transformation that reinterprets transcendental equation $\sqrt[3]{p^2-q^2}+2\sqrt{r^2-2s^2}=121t^2$ within the framework of homogeneous ternary quadratic equation and find integral solutions of the transcendental equation under frequent patterns providing specific numerical examples.

Keywords: Transcendental equation, Palindrome number, Integral solutions.

1. Introduction:

Any mathematical equation that has transcendental functions for each of its constituent variables is considered transcendental. A polynomial solution cannot represent a transcendental function, which is analytical in nature. Since the variables in these equations may be roughly known, they are easy to solve. We employ many equations that appear to have only obvious solutions in order to solve transcendental functions. However, the complexity of these equations often leads to the need for numerical methods or iterative approaches, as exact solutions may not be readily available. Consequently, mathematicians and engineers alike must rely on computational tools to approximate the values of these transcendental functions accurately. The logarithmic, exponential, trigonometric, and hyperbolic functions are examples of well-known transcendental functions, as are the opposites of each of the already mentioned functions. In addition to some analytical functions like elliptic, zeta, and gamma, there are also some surprising transcendental functions.

Fundamental concepts and ideas of number theory have been investigated in [1-4].For solving diophantine equation [5-9] has been recommended for fundamental notions and principle in number theory. Transcendental equation related thoughts and problem were collected in [10-16].
In this paper, we propose a novel methodological transformation that reinterprets transcendental equation $\sqrt[3]{p^2-q^2}+2\sqrt{r^2-2s^2}=121t^2$ within the framework of homogeneous ternary quadratic equation and find integral solutions of the transcendental equation under frequent patterns providing

specific numerical examples.

2. Method of Analysis:

The transcendental equation to be solved is $\sqrt[3]{p^2-q^2}+2\sqrt{r^2-2s^2}=121t^2$ (1)

Introducing the linear transformation,

$p=\beta^3-\alpha^2\beta, q=\alpha\beta^2-\alpha^3, r=\alpha^2+2\beta^2, s=2\alpha\beta$ (2)

Then (1) reduces to homogenous ternary quadratic form as

$\alpha^2-3\beta^2=121t^2$ (3)

Pattern 1:

Set $t=\gamma^2+\delta^2$, provided $\gamma,\delta>0$ (4)

Write 121 as,

$121=(13+4\sqrt{3})(13-4\sqrt{3})$ (5)

Using (3), (4), (5) and applying factorization method, one may get

$(\alpha+\sqrt{3}\beta)(\alpha-\sqrt{3}\beta)=(13+4\sqrt{3})(13-4\sqrt{3})(\gamma+\sqrt{3}\delta)^2(\gamma-\sqrt{3}\delta)^2$

Equating like terms, the value of α and β are

$\alpha=\alpha(\gamma,\delta)=13\gamma^2+24\gamma\delta+39\delta^2$

$\beta=\beta(\gamma,\delta)=4\gamma^2+26\gamma\delta+12\delta^2$

Therefore the required non-zero integral solutions are

$p=-612\gamma^6-5642\gamma^5\delta-15924\gamma^4\delta^2-31252\gamma^3\delta^3-4777\gamma^2\delta^4-50778\gamma\delta^5-16524\delta^6$

$q=-1989\gamma^6-9080\gamma^5\delta-26585\gamma^4\delta^2-52080\gamma^3\delta^3-79755\gamma^2\delta^4-81720\gamma\delta^5-53703\delta^6$

$r=201\gamma^4+1040\gamma^3\delta+3134\gamma^2\delta^2+3120\gamma\delta^3+1809\delta^4$

$s=104\gamma^4+868\gamma^3\delta+1872\gamma^2\delta^2+2604\gamma\delta^3+936\delta^4$

γ	δ	α	β	p	q	r	s	t	LHS	RHS
1	1	76	42	-168504	-304912	9304	6384	-2	484	484
2	1	139	80	-1033680	-1796019	32121	22240	1	121	121
2	2	304	168	-10784256	-19514368	148864	102144	-8	7744	7744
1	3	436	190	-29259240	-67142256	262296	165680	-26	81796	81796

Pattern 2:

Equation (3) can be written as $\alpha^2-3\beta^2=121t^2*1$ (6)

Write 121 and 1 as,

$121=(13+4\sqrt{3})(13-4\sqrt{3}), 1=(2+\sqrt{3})(2-\sqrt{3})$ (7)

Using (4), (6), (7) and applying factorization method, one may get

$(\alpha+\sqrt{3}\beta)(\alpha-\sqrt{3}\beta)=(13+4\sqrt{3})(13-4\sqrt{3})(\gamma+\sqrt{3}\delta)^2(\gamma-\sqrt{3}\delta)^2(2+\sqrt{3})(2-\sqrt{3})$

Equating like terms, the value of α and β are

$\alpha=\alpha(\gamma,\delta)=38\gamma^2+126\gamma\delta+114\delta^2$

$\beta = \beta(\gamma,\delta) = 21\gamma^2 + 76\gamma\delta + 63\delta^2$

Therefore the required non-zero integral solutions are

$p = -21063\gamma^6 - 210292\gamma^5\delta - 886851\gamma^4\delta^2 - 2029352\gamma^3\delta^3 - 2660553\gamma^2\delta^4 - 1892628\gamma\delta^5 - 568701\delta^6$

$q = -38114\gamma^6 - 368970\gamma^5\delta - 1531210\gamma^4\delta^2 - 3486420\gamma^3\delta^3 - 4593630\gamma^2\delta^4 - 3320730\gamma\delta^5 - 1029078\delta^6$

$r = 2326\gamma^4 + 15960\gamma^3\delta + 41384\gamma^2\delta^2 + 47880\gamma\delta^3 + 20934\delta^4$

$s = 1596\gamma^4 + 11068\gamma^3\delta + 28728\gamma^2\delta^2 + 33204\gamma\delta^3 + 14364\delta^4$

γ	δ	α	β	p	q	r	s	t	LHS	RHS
1	2	746	425	-159753675	-280414686	917766	634100	-11	14641	14641
2	1	518	299	-53497977	-92682114	447126	309764	1	121	121
2	3	1934	1170	-2784002049	-4863830138	6191254	4281876	-23	64009	64009
1	3	834	480	-223274880	-387940104	1156356	800640	6	4356	4356

Pattern 3:

Write 121 and 1 as,

$$121 = (14+5\sqrt{3})(14-5\sqrt{3}), 1 = (7+4\sqrt{3})(7-4\sqrt{3}) \quad (8)$$

Using (4), (6), (8) and applying factorization method, one may get

$$(\alpha+\sqrt{3}\beta)(\alpha-\sqrt{3}\beta) = (14+5\sqrt{3})(14-5\sqrt{3})(\gamma+\sqrt{3}\delta)^2(\gamma-\sqrt{3}\delta)^2(7+4\sqrt{3})(7-4\sqrt{3})$$

Equating like terms, the value of α and β are

$\alpha = \alpha(\gamma,\delta) = 158\gamma^2 + 546\gamma\delta + 474\delta^2$

$\beta = \beta(\gamma,\delta) = 91\gamma^2 + 316\gamma\delta + 273\delta^2$

Therefore the required non-zero integral solutions are

$$p = -1518153\gamma^6 - 15739012\gamma^5\delta - 68052621\gamma^4\delta^2 - 157084232\gamma^3\delta^3 - 204157863\gamma^2\delta^4 - 147651108\gamma\delta^5 - 40990131\delta^6$$

$$q = -2635914\gamma^6 - 27282710\gamma^5\delta - 117851410\gamma^4\delta^2 - 271946220\gamma^3\delta^3 - 353554230\gamma^2\delta^4 - 245544390\gamma\delta^5 - 71169678\delta^6$$

$r = 41526\gamma^4 + 287560\gamma^3\delta + 746984\gamma^2\delta^2 + 862680\gamma\delta^3 + 373734\delta^4$

$s = 28756\gamma^4 + 199228\gamma^3\delta + 517608\gamma^2\delta^2 + 597684\gamma\delta^3 + 258804\delta^4$

γ	δ	α	β	p	q	r	s	t	LHS	RHS
1	1	1178	680	-629193120	-1089984552	2312484	1602080	-2	484	484
2	1	2198	1269	-4087249767	-70791412914	8051926	5578524	1	121	121
1	3	6062	3496	-85742294688	-148675569336	61191876	42385504	-26	81796	81796
3	4	15558	8979	-1449470299617	-2511511184034	403296246	279390564	-39	184041	184041

3. Conclusion:

In this study, we have introduced a novel methodological approach to reinterpret a complex transcendental equation $\sqrt[3]{p^2-q^2} + 2\sqrt{r^2-2s^2} = 121t^2$ through the lens of a homogeneous ternary quadratic framework. By applying linear transformation, we successfully derived integral solutions and identified specific recurring numerical patterns, particularly involving palindrome numbers. These findings not only offer a deeper insight into the structural behavior of transcendental equations but also open up new avenues for further mathematical exploration in number theory and algebraic transformations. Future work may focus on generalizing this approach to broader classes of transcendental and Diophantine equations.

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