

Modelling Data Sets of Doppler Ultrasound Technique in Medical Diagnosis in Determining Accurate Velocity Measurements of Blood Flow in Patients

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Abstract

Doppler ultrasound technique is one of the most frequently used Sonography methods in medical diagnosis in diagnosing blood flow measurements / fluctuations in patients. Modelling of such kind of data is necessitated for accurate periodic observations and predictions of blood flow. Monitoring blood flow fluctuations in patients is to be done periodically for accurate observations and predictions. A Fourier series method is suggested for accurate predictions of blood flow in vessels, organs and also to detect blood clots, and other obstacles in vessels.

Keywords: Blood Clots, Blood Flow, Blood Vessels, Doppler ultrasound Technique, Fourier Series Method.

1. Introduction

In regular medical diagnosis practices, ultrasound technique, also called as Sonography, is used for many medicinal purposes including Pregnancy monitoring, diagnostic imaging, guiding procedures, and also for specific applications. It has many other industrial applications too. Ultrasound technique uses high-frequency sound waves to create real-time images of internal structure of organs. It is a safe imaging technique without having any kind of side effects / negative impacts on human bodies. Its major advantages include a non-invasive, non-ionizing, real-time imaging and relatively inexpensive method as well. Its specific applications include the following:

- **Pregnancy Monitoring:** Ultrasound is crucial for monitoring fetal development during pregnancy.
- **Guiding procedures:** ultrasound guidance is used to place needles accurately for biopsies / injections / fluid aspiration.
- **Therapeutic applications:** Ultrasound can be used for breakup kidney stones, in treating tissue injuries and also used in the treatment of tumors.

- Doppler ultrasound: It measures blood flow velocity which is helpful in diagnosing vascular conditions like blockages / narrowed arteries. A very popularly used technique in measuring blood flow in vessels.
- Diagnostic imaging: Ultrasound is used to visualize the structures of internal organs / blood vessels / soft tissues. This includes imaging echocardiography (heart), abdomen (liver, gallbladder etc.) musculoskeletal system (muscles and joints) and reproductive organs.
- Ultrasonic sensors: They have many applications in day-to-day related activities.

2. Doppler Effect

The Doppler effect, in Physics, also known as Doppler Shift, is the change in the frequency of a wave in relation to an observer who is moving relative to the source of the wave.

$$f_0 = (v + v_0) f_s / (v + v_s) \quad \text{-----} \quad (1)$$

where, f_0 = observer frequency of sound

v = speed of sound waves

v_0 = observer velocity

v_s = source velocity

f_s = actual frequency of sound wave

Doppler shift in physics has many practical applications including Medical Imaging and Speed Measurement. Here, the focus of the paper is on medical diagnosis.

2.1 Medical Imaging

* Blood Flow Measurement: Doppler Shift can be used to measure blood flow velocity in various parts of the body organs and also used in vascular diseases. In recent practices, this technique has been using to measure blood flow pumping volume in Liver and other small important organs.

2.2 Echocardiography

* Doppler shift can be used to find blood flow volume within the heart. It is also important for measuring the distribution of blood outflow to all other nerves connected to the heart.

2.3 Doppler ultrasound

Its underlined mechanism uses Doppler shift which evaluates the change in frequency of a wave when there is a relative motion between the source and the receiver. In medical ultrasound, this implies bouncing sound waves off moving red blood cells and analyzing the frequency shift of the returning echoes to determine blood flow velocity and direction.

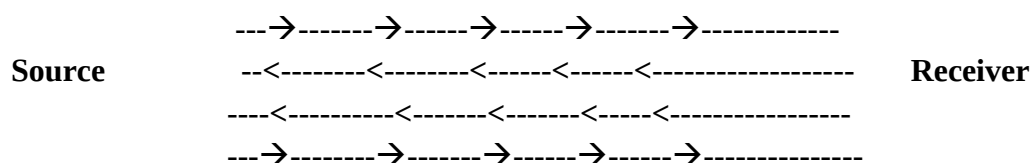


Fig. 2.3.1 Normal blood flow with normal frequency shift

Ultrasound waves are sent from the source to the receiver. If these waves are bouncing back off from the source without any major frequency shift then it is observed that there is no change in the relative speed of the blood flow. It concludes that blood flow is normal.

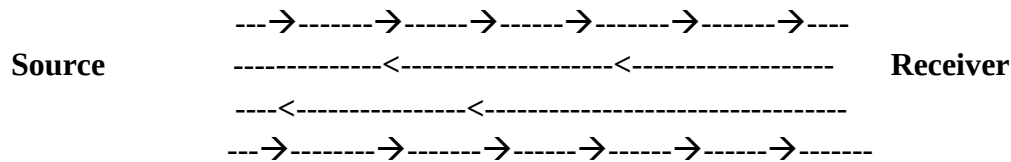


Fig. 2.3.2 blood flow with problems in frequency shift

In Doppler's ultrasound application, the transducer emits sound waves that penetrates into the body and reflects off moving red blood cells. It helps in observing the normal movement of red blood cells in blood flow between organs.

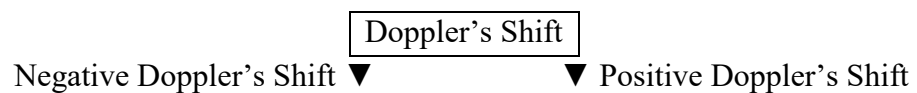


Fig. 2.3.3 Two variations of Doppler's shift

If blood cells are moving towards transducer, the reflected echo sound waves will have a higher frequency, which is called a positive Doppler's Shift.

If blood cells are moving away transducer, the reflected echo sound waves will have a lower frequency, which is called a negative Doppler's Shift.

2.4 Doppler Angles

The angle between the ultrasound beam and the direction of the blood flow (Doppler's angle) influences the accuracy of velocity measurements. If the ultrasound beam is parallel to the blood flow gives the most accurate velocity observation, which is called a zero-degree angle. Angles greater than 60° can lead to significant underestimation of velocity of blood. Also known as "Insonation angle".

2.5 Types of Doppler ultrasound

- * Spectral Doppler ultrasound: It presents the Doppler's information as a waveform allowing for a detailed analysis of blood flow characteristics.
- * Color Doppler Ultrasound: It uses color to represent blood flow velocity and direction on a ultrasound image providing a 2D visual map of blood flow.
- * Pulsed wave Doppler ultrasound: It allows for velocity measurements at a specific deeper depth, very useful for deeper vessels. It is used only in some specific cases.
- * Continuous wave Doppler ultrasound: it provides velocity information without depth resolution, primarily used for superficial structures. It also uses in adult cardiac scanners to observe high velocities in aorta.

2.6 Evaluating Doppler Frequency

It is calculated by $f_d = (2 * B_t * V * \cos \Theta) / C$, where ----- (2)

f_d = Doppler Shift

B_t = Transmission beam

V = blood velocity

Θ = angle of incidence between the ultrasound beam and the direction of the blood Flow

C = speed of sound in tissue

Higher Doppler frequency can be obtained, if;

- Velocity is increased
- Beam is more aligned in the direction of blood flow
- Higher frequency is used
- Higher-frequency Doppler signal is obtained if the beam is aligned more to the direction of the blood flow

Filters are used in all kinds of Doppler ultrasound devices to cut out high amplitudes, if any. Low-frequency Doppler signals are generated due to vessels wall motion. For a minimum requirements of Filter frequencies, users can exclude frequencies below 50 Hz, 75 Hz, 100 Hz etc., Thus frequency limits can be altered by filters as per the needs and requirements of the case.

2.7 Aliasing

Maximum Doppler frequency (without any ambiguity) can be measured as half the pulse repetition frequency. Combining the blood velocity and beam angle provides more half the pulse repetition frequency, which is called Aliasing.

More Doppler frequency = (pulse repetition frequency)/2 → gives unambiguous results

If Doppler frequency > (pulse repetition frequency)/2 → gives ambiguous results
(blood flow + beam angle)
----- (3)

2.8 Beam Angles

It is defined as the angle where the sound pressure level of the beam decreases by 3 dB on either side of the peak sound pressure level. Beam angles will have their impact on Doppler shift. There are 2 ways of utilizing beam angles.

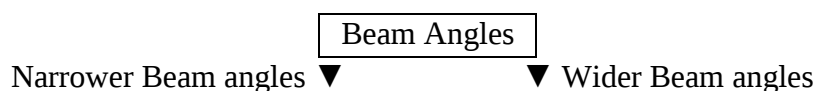


Fig 2.8.1 Two types of Beam angles used in medical diagnosis

- Narrower beam angles: are used for high accuracy and precision requirements. Also used in distance measurements.
- Wider beam angles: uses for detecting obstacles over a larger area.

- Choosing the correct beam angle: it depends upon the requirements of the patient's condition. A sensor with a 15-degree beam angle is suitable for precision measurements, while a sensor with a 60-degree beam angle is suitable for obstacle detection over wider areas.

2.8.1 Cosine angle

The Doppler equation says that the measured frequency shift (calculated velocity of blood flow) is dependent on the cosine of the Isolation angle. It is also known as angle dependence.

3. Modelling of Doppler's ultrasound waves (Signals)

Doppler's ultrasound waves are signals with frequencies. Examining Doppler's ultrasound observations on patients may be difficult in all types of situations. Instead, a mathematical modelling of ultrasound observations of a particular patient, whose health condition is in very critical situation, is very much needed to predict the patient's health condition from time to time. Here, a Fourier Series method of formulating a patient's Doppler ultrasound data into a periodic wave form (signal) is attempted.

Sometimes the function is not given by a mathematical model of a particular signal, but given by a graph or a table of corresponding values, then the process of finding Fourier Series of a function given by such values of the function is known as Harmonic Analysis. Here, let the wave form (signal) can be represented by a mathematical function $f(x)$, where 't' is an independent variable represents time. Let the time period be $[-\Pi, \Pi]$. Fourier Series of a signal is given by:

$$f(x) = (0.5) a_0 + \sum_{(n=1 \text{ to } \infty)} (a_n \cos nx + b_n \sin nx) \text{ ----- (4)}$$

$$\text{Where } a_0 = (1/\Pi) \int f(x) * dx, (x = 0 \text{ to } 2\Pi) \text{ ----- (5)}$$

$$\text{i.e., } a_0 = 2 [\text{Mean value of } f(x) \text{ in } [0, 2\Pi]] \text{ ----- (6)}$$

[Since Mean = $1/(b-a) * \int f(x) dx$, ($x = 0$ to 2Π), where a & b are lower & upper limits respectively]

$$a_n = (1/\Pi) \int f(x) * \cos nx * dx, (x = 0 \text{ to } 2\Pi) \text{ ----- (7)}$$

$$\text{i.e., } a_n = 2 [\text{Mean value of } f(x) * \cos nx, (x = 0 \text{ to } 2\Pi)]$$

$$b_n = (1/\Pi) \int f(x) * \sin nx * dx, (x = 0 \text{ to } 2\Pi) \text{ ----- (7)}$$

$$\text{i.e., } b_n = 2 [\text{Mean value of } f(x) * \sin nx, (x = 0 \text{ to } 2\Pi)] \text{ ----- (8)}$$

In this Harmonic Analysis,

- Term $a_1 \cos x + b_1 \sin x$ is called Fundamental or First Harmonic
- Term $a_2 \cos 2x + b_2 \sin 2x$ is called Second Harmonic and so on

The following data are collected from one of the popular Medical Diagnostic centers in Hyderabad in order to model data sets of Doppler ultrasound technique to find blood flow measures of a patient. Even though a big data set are collected, here it is not possible to present the entire data set. Instead, a part of the data set are presented. The values of x and $f(x)$ are present in the following table 3.1.

Table 3.1 Data sets are collected from beam of Doppler ultrasound diagnosis

s. no.	x(in degrees)	f (x)	s. no.	x(in degrees)	f (x)
1	0	2.34	7	210	0.83
2	30	3.01	8	240	0.51
3	60	3.69	9	270	0.88
4	120	4.15	10	300	1.09
5	150	3.69	11	330	1.19
6	180	2.20	12	360	1.64

From the collected data of the above table, a new table is to be generated for finding cosine and sin values. Here, each table contains values of first and second harmonics of the function $f(x)$ ($a_1 \cos x + b_1 \sin x$ and $a_2 \cos 2x + b_2 \sin 2x$). Calculations are done for two harmonics only.

Table 3.2 Data sets are generated for first and second harmonics

s. no.	x(in degrees)	Sin x	Sin 2x	Cos x	Cos 2x
1	0	0	0	1	1
2	30	0.50	0.87	0.87	0.50
3	60	0.87	0.87	0.50	-0.50
4	90	1.00	0	0	-1.00
5	120	0.87	-0.87	-0.50	-0.50
6	150	0.50	-0.87	-0.87	0.50
7	180	0	0	-1	1.00
8	210	-0.50	0.87	-0.87	0.50
9	240	-0.87	0.87	-0.50	-0.50
10	270	-1.00	0	0	-1.00
11	300	-0.87	-0.87	0.50	-0.50
12	330	-0.50	-0.87	0.87	0.50

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Values of the following terms are to be evaluated:

- i) $f(x) * \sin x$
- ii) $f(x) * \sin 2x$
- iii) $f(x) * \cos x$
- iv) $f(x) * \cos 2x$

Means of a_0, a_1, a_2, b_1, b_2 are as follows:

- $a_0 = 2 * \text{mean of } f(x)$
- $a_1 = 2 * \text{mean of } f(x) * \cos x$
- $a_2 = 2 * \text{mean of } f(x) * \cos 2x$
- $b_1 = 2 * \text{mean of } f(x) * \sin x$
- $b_2 = 2 * \text{mean of } f(x) * \sin 2x$

The following table 3.3 contains various multiples of $f(x)$ with sine and cosine terms up to second harmonic only.

Table 3.3 Values of $f(x)$ with sine and cosine multiples

$f(x)$	$f(x) \sin x$	$f(x) \sin 2x$	$f(x) \cos x$	$f(x) \cos 2x$
2.340	0	0	2.340	2.340
3.01	1.505	2.619	2.619	1.505
3.690	3.210	3.210	1.845	-1.845
4.150	4.150	0	0	-4.150
3.69	3.210	-3.210	-1.845	-1.845
2.20	1.100	-1.914	-1.914	1.100
0.83	0	0	-0.830	0.830
0.51	-0.255	0.444	-0.444	0.255
0.88	-0.766	0.766	-0.440	-0.440
1.09	-1.09	0	0	-1.090
1.19	-1.035	-1.035	0.595	-0.595
1.64	-0.820	-1.427	1.427	0.820
Total → 25.22	9.209	-0.547	3.353	-3.115

Evaluating mean values:

$$a_0 = 2 * \text{mean of } f(x) \\ = 2 * 25.22 / 12 = 4.203 \text{ ----- (9)}$$

$$a_1 = 2 * \text{mean of } f(x) * \cos(x) \\ = 2 * 3.353 / 12 = 0.559 \text{ ----- (10)}$$

$$a_2 = 2 * \text{mean of } f(x) * \cos 2x \\ = 2 * (-3.115) / 12 = -0.519 \text{ ----- (11)}$$

$$b_1 = 2 * \text{mean of } f(x) * \sin x \\ = 2 * 9.209 / 12 = 1.535 \text{ ----- (12)}$$

$$b_2 = 2 * \text{mean of } f(x) * \sin 2x \\ = 2 * (-0.547) / 12 = -0.091 \text{ ----- (13)}$$

Substituting the above values of (9) to (13) in the equation (4), then

$$f(x) = (0.5) a_0 + a_1 \cos x + a_2 \cos 2x + b_1 \sin x + b_2 \sin 2x + \text{-----}$$

The Fourier Series modelling of Doppler's ultrasound wave form is:

$$f(x) = 2.1015 + 0.559 \cos x - 0.519 \cos 2x + \text{-----} \\ + 1.535 \sin x - 0.091 \sin 2x + \text{-----}$$

4. Conclusion

Mathematical modelling of wave forms will be very useful in assessing the day-to-day monitoring of patients. If models of signals are available the they can be modelled by using Fourier series which has



been a very powerful tool in designing of periodical signals. If direct models of signals / waveforms are not available, the Harmonic Analysis, with the help of available data, helps a lot in designing mathematical models approximately. The approximation value can be made more accurate if the estimations could be done at the level of fourth and fifth harmonics levels. Accuracy of the modelling increases linearly as the level of harmonic evaluation increases accordingly.

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