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Discretionary Lane Changing decision for Avoiding the Rear Crash Using Quantal Response Equilibrium-based Game theory.

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Abstract:

Objective: This research is proposing the parametric model for this kind of lane changing drivers who are boundedly rational in the conflicting position on Discretionary Lane Changing (DLC) situation.

Methods: Quantal response equilibrium-based game theory model are employed to address the bounded rational behavior of driver, to find out the parameter value of the model, and to propose the probabilistic decisions of DLC drivers for conflicting situations. As a result, this calibration model uses 30 lane-changing events of the Dhaka-Tangail highway of Bangladesh dataset to suggest DLC driver behavior. Additionally, this model validates 16 Lane Changing (LC) events of the same dataset. Then, the model are calibrated and collected to check the test dataset. The confusion matrix are used to compare between decisions of this real dataset and decision model predicted.

Finding: The model accurately predicts about overall 75% of actions for this research data in particular, it predicts 74% of TRV activities during the LC permission scenario. The false-alarm-rates for LC, Non-LC, yielding and forbidding decisions are 15.12%, 0.00%, 35.00% and 8.33%, respectively using the calibration approach.

Novelty: Therefore, the predictions of LC decision with calibrated parameters were found to be quite accurate to demonstrate the model's robustness.

Keywords: Discretionary Lane Changing, Bi-level programming, Genetic Algorithm, Quantal Response Equilibrium, Game Theory.

1. Introduction:

When a driver of a vehicle on an urban roadway needs to travel in safety in the Lane Changing (LC) decision, which is a very difficult and contradictory activity. DLC behaviors frequently cause crashes near freeway off-ramps. Many studies over the past two decades have used the binary decisions-based lane-changing model to try to overcome this issue action. The game theory can provide suggestions to many decision makers [1].

The grade of interaction problem justifies the scientific analysis of decision-making between two vehicle systems. Interactive mathematical models are used in the decision modeling of vehicle system maneuver [2]. However, most of the principal mathematical models have either one-dimensional or one-sided interaction between conflicting drivers or another driver. For example, gap acceptance models search the critical gap at the target-lane for SV, and CFMs accept only FV movements for SV movement decisions. Here conflicting drivers do not accept the interactive decision of other drivers.

The DLC is necessary for drivers to be able to arrive at the planned purpose, while the latter is freely adopted by drivers for enlightening their driving situations. DLC, related to MLC, is more complex because drivers frequently need to estimate its necessity, interest, and safety, while for MLC, drivers typically only need to estimate its safety [3]. Game theory is a promising evaluation tool in the vehicle communicating driving systems, such as vehicle-to-infrastructure and vehicle-to-vehicle communications. To estimate the efficiency and safety impacts of traffic infrastructure, game theory is a suitable for contributing to the cost-benefit studies.

Game theory enables the multilateral analysis as unique insights quantitatively, namely, behavioral norms and moral hazards of interactions. Driver equilibrium behaviors are established by behavioral norms of interactions between SV driver and Target Rear Vehicle (TRV) driver. The formal road guidelines and the behavioral norms of interactions may vary when equilibrium comes endogenously. The behavioral norms can be described numerically as the driving framework by using game theory [4], [5]. The decision payoffs of drivers are formed from a numerical framework. The behavioral norms of interactions are useful to the payoffs of its equilibrium behavior.

When the traffic rules change by driver behavior protection to improve traffic safety, moral hazards are created during driver interaction [6]. These moral hazards might complicate driver anticipation, which reduces the safety of other drivers. Moral hazards deteriorate the advantage of the safety rule that the authority rigidly enforces. Therefore, authority-given traffic safety rules may not decrease the rear crash in urban roadways. In contrast, game theory decreases moral hazards and increases behavioral norms to arrive at the equilibrium point of interaction.

However, relatively few studies have used the Nash equilibrium-based game theory model as an at least four decision-based model to solve this kind of problem. Although the game's participants' (the drivers of the target rear car and the lane-changing vehicle in the urban traffic system) actions are limitedly reasonable when they interact. Current studies use a Nash-equilibrium game theory model that takes into account the entire range of rational driver behavior. The forced lane-changing decision was introduced in a landmark work [7], that utilized the QRE model based on bounded rational behavior.

QRE allows the solution point for inaccurate information by stochastic payoff function [8]. QRE outputs generate probabilities from quantal response functions wherein probabilities arise stochastically in 0 and 1. Therefore, as binary decision models, probit and logit models include the quantal response functions as a probabilistic decision approach [9]. Quantal response functions provide significantly better decisions than other functions. However, the best payoff is sometimes avoided for QRE. Although, this equilibrium may contain some errors based on the belief, however by some iterations, these errors may decrease using quantal response functions. Therefore, the fitted decision is equal to the average probabilistic decisions. In this manner, the stochastic form of NE is QRE, where the opponent player deterministic anticipations are equal to their probability decisions [10].

The QRE model uses for the DLC probabilistic decision where the driver can make the interaction decision. This model is a stochastic form of NE because the inaccurate information of driver decision may affect the opponent payoffs. Then, the opponent drivers may not properly appraise road stimuli. Driver anticipations and choice-probabilities are inscribed in payoffs by using the quantal response functions. When average anticipation is equal to real maneuver probability, QRE converges at the solution point.

Driver anticipations and perception-errors depend on the quantal response function in QRE model. Therefore, the logit QRE is more realistic for this research approach.

Few studies have used the Nash equilibrium-based game theory model as an at least four decision-based model to solve conflicting situation. Although, the drivers are not fully rational, so the problem should be solved by the Quantal Response Equilibrium (QRE) model. Furthermore, no study applied the QRE model with bounded rational behavior to DLC-based probabilistic decisions for conflicting situations. This research is to use out a Quantal Response Equilibrium (QRE) model for DLC action based on bounded rational behavior, and to propose the probabilistic decisions of the LC vehicle (following vehicle) and target following vehicle, and to find the conflicting situations in the DLC decision. The robust model can suggest the LC decision for safe travel in the next generation autonomous vehicles.

2. Methodology:

Drivers can decide by lowest perceived errors in QRE-based game model [11]. They can also take LC decision using bounded-rational behavior in this model. Two players game is shown in Table 1 that each driver at lane merging situation must choose one decision from above-mentioned two decisions. Besides, the LC driver has two options; one is LC, and another is non-LC. Therefore, four possible outcomes are created for every player from this game. Equations (1) to (4) calculate the utilities of payoffs. The decision of one player depends upon the decision of another player. The payoff values are mainstream in Expected Utility (EU) functions to create the influencing decision.

Table 1. Payoff matrix for the two players game [12].

		TRV decision	
		Yield	Forbid
SV decision	LC	$[\alpha_1 SVv], (\beta_1 TRV\ v)$	$[\alpha_1 SVv], (\beta_2 TRV\ v)$
	Non-LC	$[\alpha_2 CAg], (\beta_1 SV\ v)$	$[\alpha_2 CAg], (\beta_2 TRVv)$

EU values for player 1:

$$EUlc = (py + (1 - py)) \times (\alpha_1 SVv), \quad (1)$$

$$EUnlc = (py + (1 - py)) \times \alpha_2 CAg. \quad (2)$$

EU values for player 2:

$$EUy = plc \times \beta_1 TRV\ v + (1 - plc) \times \beta_1 SVv, \quad (3)$$

$$EUny = (plc + (1 - plc)) \times (\beta_2 TRVv). \quad (4)$$

where, plc refers LC probability of player 1, and py refers give-way probability of player 2

2.1 Logit quantal response equilibrium model

The strategy set of any driver depends upon another driver actions. EU functions as quantal response functions estimate the decision probabilities by equations (5) and (6) [10], [13].

$$p_{lc} = \frac{e^{\lambda EU_{lc}(p_y)}}{e^{\lambda EU_{nlc}(p_y)} + e^{\lambda EU_{lc}(p_y)}} \quad (5)$$

$$p_y = \frac{e^{\lambda EU_y(p_{lc})}}{e^{\lambda EU_{ny}(p_{lc})} + e^{\lambda EU_y(p_{lc})}} \tag{6}$$

where, λ refers to the rational behavior in driving that it significantly impacts to the driver perceptions behavior. In equations (5) and (6), $\lambda = 0$ and $\lambda = \infty$ indicate the irrational behavior and rational behavior of driver, respectively [10]. The results of the model calibration provide a method for more naturally taking driver variety into account. As a result, all models produce improved explanatory power for actual lane-changing behaviors when LC trajectory factor are included [14]. The Figure 1 provided the calibration of drivers probabilistic decisions.

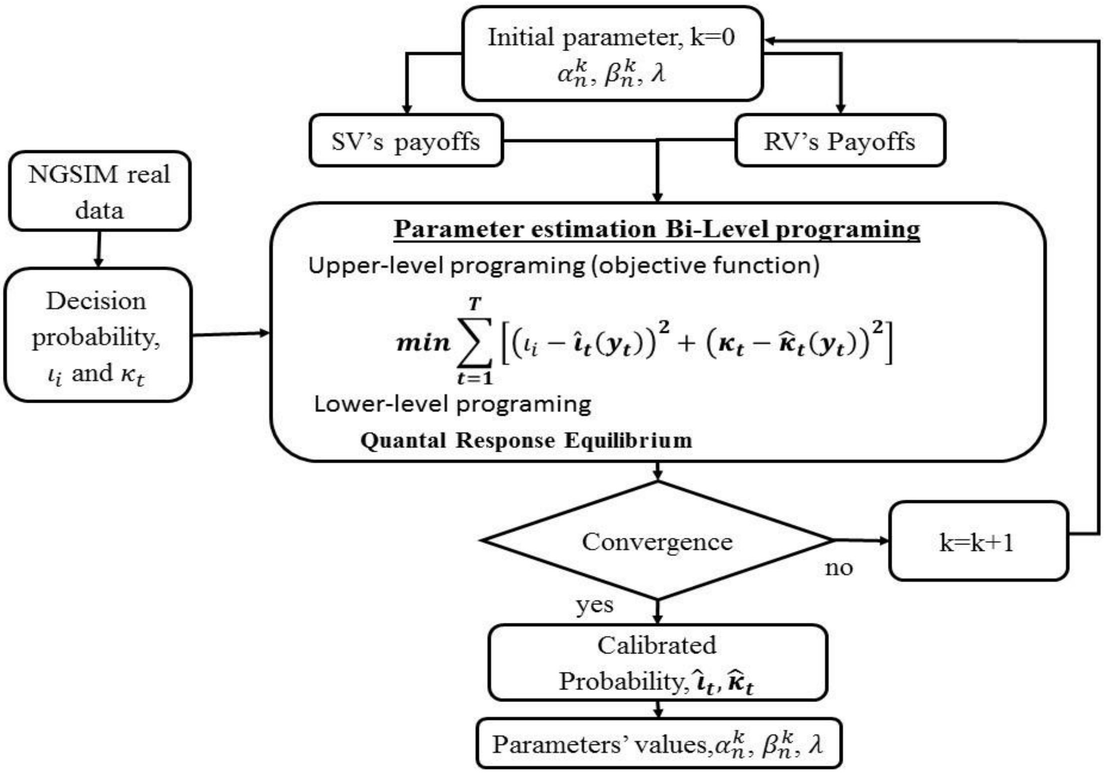

Figure 1. Calibration framework (bi-level programming) [15].

3. Results and Discussion

This part presents the data information, data processing and statistical analyzes of collected factors. Data information includes the data collection area information, vehicle trajectory information, and data size and data collection time. Besides, data processing provides the data to use in methods.

All the data collected in this research is from the Dhaka-Tangail Highway. The data collection area and trajectories are shown in Figure 2. Figure 2 with four lanes. Except for one auxiliary lane, the other three lanes are designated as main traffic for highway vehicles where DLC situations occur. Most of the vehicles plying on the Dhaka-Tangail highway have a speed of between 60 and 70. The data is collected by making videos using high-quality cameras [16]. This study has been classified into four categories (DLC, MLC, Non-LC, and following vehicles). DLC and mandatory lane change are LC movements; Non-LC is a driving movement in which the driver wants to change lanes but may not change lanes this time. Whether a bus changes lanes or not depends on SV and TRV. SV can change lanes if the target vehicle permits.

(a) Target Lane Change (b) Attempt to change lanes **(c)** Lane Changing Prosses.

Figure 2. LC scenarios.

3.1 Explanations:

Two lanes in each direction of a four-lane highway were planned. It was roughly 1 km long and had a posted speed restriction of 100 km/h. In the experiment, the existing driving lane was blocked and lane switching was required. The game theory technique required SV to switch lanes while FVs were programmed to speed up, slow down, or not be influenced by SV's attempts to merge. (Take note that FVs are programmed for data collection purposes, and their behavior is considered the same as that observed in the database; prior knowledge of programmed vehicles is not used for LCD modeling.) Since the roadway segment consisted of two lanes in each direction to avoid complexity in designing vehicular interactions, the lane-changing maneuver of FV when SV is merging is very unlikely. Thus, this strategy is not observed in simulator data. The vehicular interaction and the design of connectivity are explained below.

When the speed of Hanif Paribahan is 16.67 m/s and the speed of Ovi Paribahan is 15.83 m/s, Hanif Paribahan cannot change the lane even after trying hard. There will be no lane changes at this time. Even if Hanif Paribahan increases his speed to 18.06m/s to change the lane and Ovi Paribahan keeps his speed unchanged, then the two vehicles will stay parallel. Keeping the speed same and Ovi's car reducing its speed slightly to 15.28 m/s, they will also be in the same position. If Hanif Paribahan increases its speed to 18.06 and Ovi Paribahan reduces its speed to 15.28 m/s, then Hanif Paribahan Of course, the lane will change.

Table 2 below discusses what can happen when Hanif Paribahan wants to change lanes with Ovi Paribahan on a four-lane road on the Dhaka-Tangail Highway in Bangladesh:

Table 2 : lane changing incidents between Ovi and Hanif Travels.

Player I

			Hanif	
			Speed, 16.67	Speed, 18.06
Player II	Ovi	Speed, 15.83	No LC	Parallel
		Speed, 15.28	Parallel	Sure LC

Table 3 below discusses what can happen when National Travels wants to change lanes with S.I. Paribahans on a four-lane road on the Dhaka-Tangail Highway in Bangladesh:

When the speed of National Travels is 17.25 m/s and the speed of S.I. Paribahan is 16.03 m/s, National Travels cannot change the lane even after trying hard. There will be no lane changes at this time. Even if National Travels increases its speed to 19.20 m/s to change the lane and S.I. Paribahan keeps his speed unchanged, then the two vehicles will stay parallel. Keeping the speed same and S.I. Paribahan reducing his speed slightly to 14.28 m/s, they will also be in the same position. If National Travels increases its speed to 19.20 and S.I. Paribahan reduces its speed to 14.28 m/s, then National Travels, of course, the lane will change.

Table 3 : : lane changing incidents between SI and Hanif Travels.

Player I

			Hanif	
			Speed, 17.25	Speed, 19.20
Player II	SI	Speed, 16.03	No LC	Parallel
		Speed, 14.28	Parallel	Sure LC

The table 4 below discusses what can happen when National Travels wants to change lanes with S.R. Paribahan on a four-lane road on the Dhaka-Tangail Highway in Bangladesh:

When the speed of National Travels is 17.25 m/s and the speed of S.R. Paribahan is 16.03 m/s, National Travels cannot change the lane even after trying hard. There will be lane changes at this time. Even if National Travels decreases its speed to 16.20 m/s to change the lane and S.R. Paribahan keeps his speed unchanged, then the two vehicles will stay parallel. Keeping the speed, the same and S.R. Paribahan increasing its speed slightly to 16.78 m/s, there will also be no lane change. If National Travels decreases its speed to 16.20 and S.R. Paribahan increases its speed to 16.78 m/s, then National Travels cannot lane change.

Table 4 : : lane changing incidents between SR and National Travels.

	Player I				
			National		
			Speed, 17.25	Speed, 16.20
Player II	SR	Speed, 14.03	No LC	Parallel
		Speed, 16.78	Parallel	Sure LC

As above discussion, the situation can happen when Akota -Paribahan wants to change lanes with Manik-Paribahan on a four-lane road on the Dhaka-Tangail Highway in Bangladesh.

When the speed of Akota Paribahan is 13.25 m/s and the speed of Manik Paribahan is 13.03 m/s, Akota Paribahan cannot change the lane even after trying hard. There will be no lane changes at this time. Even if Hanif Paribahan increases his speed to 15.76m/s to change the lane and Manik Paribahan keeps his speed unchanged, then the Akota Paribahan must be lane change. If Akota Paribahan and Manik Paribahan do not change their speed, then there will no lane change. Even if Manik Paribahan does not change his speed, Akota Paribahan must change lanes if he increases his speed to 15.76.

3.2 Model Calibration:

This game theory model uses payoff values with instances from the dataset that include lane changes to suggest the DLC driver behavior. Additionally, this model uses instances from the trail dataset that change lanes. By employing a validation test, this study determines the rates of false alarm for the model's lane-changing decision for the subject vehicle, non-lane-changing decision for the subject vehicle, yielding decision for the target rear vehicle, and forbidding decision for the target rear vehicle.

Additionally, this study offers the calibrated parameters suggestions for modifying the dynamic factors in conflicting situation as shown the Table 5. Future high-performing transportation simulation software may employ the improved model to reduce accidents and bottlenecks.

Table 5. Calibrated parameters included value.

Utility	Para.	Calibration Value	Total case	False Alarm Rate %
SV	α_1	0.92	6	15.12%,
	α_2	0.55	10	0.00%,
TRV	β_1	0.01	8	35.00%
	β_2	0.23	8	8.33%,

Figure 3 and Figure 4 explain the probabilities of LC and LC permission for SV and TRV respectively. When the LC of SV is high, and TRV has the probability of not permission for LC then this situation creates the conflict or risk LC. The false-alarm-rates for LC, Non-LC, yielding and forbidding decisions are 15.12%, 0.00%, 35.00% and 8.33%, respectively using the calibration approach, and 10.81%, 0.00%, 36.36% and 40.00%, respectively using the validation test.

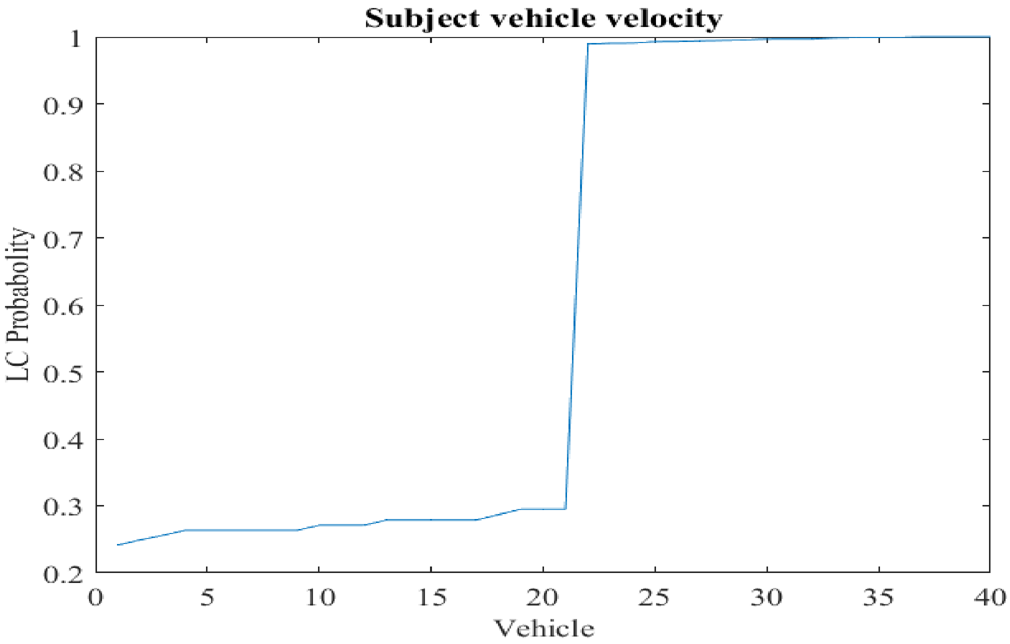

Figure 3: LC probability for Subject vehicle velocity.

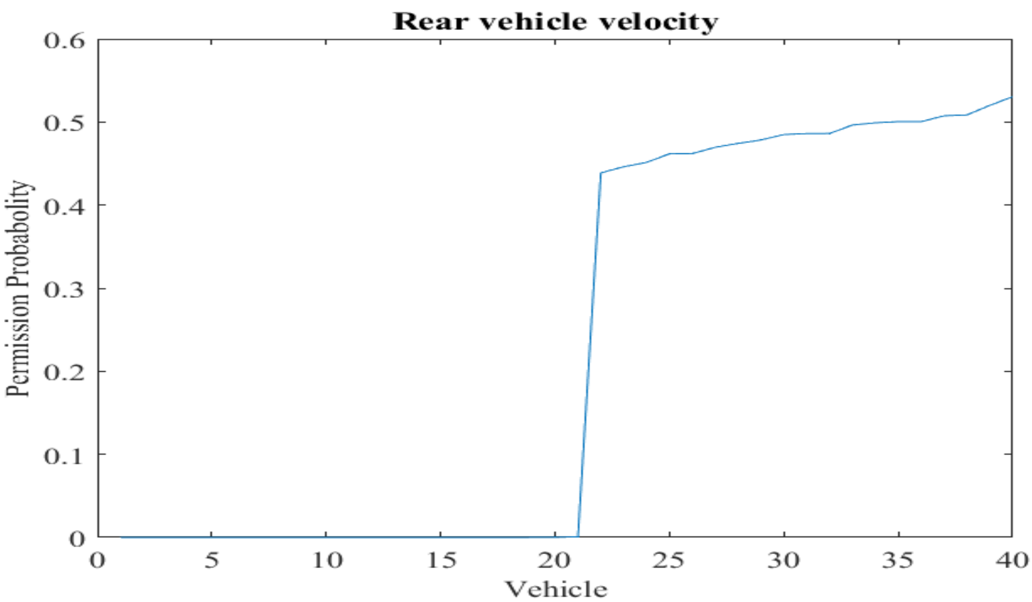

Figure 4: Permission probability for Rear vehicle velocity.

3.3 Model Validation:

This study not only verifies the acts of the LC decisions of SV, but it also validates the activities of TRV. The model's predicted behaviors of TRV (LC permission and not permission) are contrasted with the observed TRV behavior (in terms of acceleration). By using SV and TRV confusion matrix, the comparison of this research and other researches [17] and [8], is displayed in Table 6. The model accurately predicts about overall 75% of actions for this research data in particular, it predicts 74% of TRV activities during the LC permission scenario. For the references [17] and [8], the overall false alarm rates are roughly 17% and 17.83% respectively, whereas the overall false alarm rates of this research is

14.63%. The model's prediction accuracy for TRV action can be interpreted in a manner similar to that of SV. For example, the model successfully predicts 74% of TRVs' decisions to accelerate during the LC permission scenario. The created model validates the SV operations during the merging event with a respectable prediction accuracy. Therefore, this research prediction is better than previous researches.

Table 6: Model comparison using the confusion matrix.

Utility	**Para.**	**False Alarm Rate % (This article)**	**False Alarm Rate % (article [17])**	**False Alarm Rate % (article [8])**
SV	LC	15.12%,	20.00%,	14.00%,
	Not LC	0.00%,	10.00%,	15.00%,
TRV	Permission	35.00%	33.00%	24.00%
	Not permission	8.33%,	5.00%,	18.33%,

4. CONCLUSION

This research includes the Quantal Response Equilibrium (QRE) approach to determine the player decision probability. The interacted players are SV and TRV wherein SV start the Discretionary Lane Changing (DLC) to avoid the rear crash during its action. The QRE model is calibrated and validated by training data (70%) and test data (30%), respectively, which are collected from lane-five and lane-four of the NGSIM (US-101, California freeway) data set. Where QRE models for LC and yielding decisions at freeway main-lane. The LC scenarios are identified using the graphical representation, where vehicle driver changes from lane-one to lane-two at Dhaka-Tangail at 11.00 am to 4.00 pm, August 2022. In this decision model, relative velocity of SV and TRV, the relative velocity of TFV and TRV, and velocity of SV and TRV are incorporated with other LC microscopic parameters during LC starting time. The payoff function of driver decision included parameters are estimated by bi-level programming as a calibration approach. In bi-level programming, GA is applied, and the probabilities of player perceptions are collected [18]. Then, the parameters are calibrated and collected to check the test dataset. The false-alarm-rates for LC, Non-LC, yielding and forbidding decisions are 15.12%, 0.00%, 35.00% and 8.33%, respectively using the calibration approach, and 10.81%, 0.00%, 36.36% and 40.00%, respectively using the validation test. Therefore, this research finding indicate that the SV velocity influences more to increase the conflict probability during LC. Hence, this research also suggests that interacted vehicles can avoid conflicting driving by controlling such trajectories that influence more conflict probability.

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