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# Computation of Downhill Kepler Banhatti and Modified Downhill Kepler Banhatti Indices of Certain Chemical Structures

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**Abstract**

In this paper, we introduce the downhill Kepler Banhatti and the modified downhill Kepler Banhatti indices of a graph. Furthermore, we compute these newly defined downhill Kepler Banhatti indices for some standard graphs, wheel graphs, gear graphs, helm graphs and certain chemical structures.

**Keywords:** downhill Kepler Banhatti index, modified downhill Kepler Banhatti index, structure.

## 1. Introduction

The simple graphs which are finite, undirected, connected graphs without loops and multiple edges are considered. Let $G$ be such a graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(u)$ of a vertex $u$ is the number of vertices adjacent to $u$.

A $u$-$v$ path $P$ in $G$ is a sequence of vertices in $G$, starting with $u$ and ending at $v$, such that consecutive vertices in $P$ are adjacent, and no vertex is repeated. A path $\pi = v_1, v_2, \ldots v_{k+1}$ in $G$ is a downhill path if for every $i$, $1 \leq i \leq k$, $d_G(v_i) \geq d_G(v_{i+1})$.

A vertex $v$ is downhill dominates a vertex $u$ if there exists an downhill path originated from $u$ to $v$. The downhill neighborhood of a vertex $v$ is denoted by $N_{dn}(v)$ and defined as: $N_{dn}(v) = \{u: v$ downhill dominates $u\}$. The downhill degree $d_{dn}(v)$ of a vertex $v$ is the number of downhill neighbors of $v$, see [1].

In [2], Kulli introduced the Kepler Banhatti index and this index is defined as

$$KB(G) = \sum_{uv \in E(G)} \left( d_G(u) + d_G(v) + \sqrt{d_G(u)^2 + d_G(v)^2} \right).$$

Recently, some Kepler Banhatti indices were studied in [3, 4, 5, 6, 7].

Motivated by the Kepler Banhatti index, the downhill Kepler Banhatti index of a graph $G$ is defined as

$$DWKB(G) = \sum_{uv \in E(G)} \left( d_{dn}(u) + d_{dn}(v) + \sqrt{d_{dn}(u)^2 + d_{dn}(v)^2} \right).$$

We define the modified downhill Kepler Banhatti index of a graph $G$ as

$$^{m}DWKB(G) = \sum_{uv \in E(G)} \frac{1}{d_{dn}(u) + d_{dn}(v) + \sqrt{d_{dn}(u)^2 + d_{dn}(v)^2}}.$$

Recently, some downhill indices were studied in [8-16].

In this paper, the downhill Kepler Banhatti and the modified downhill Kepler Banhatti indices for some chemical structures are determined.

## 2. Results for Some Standard Graphs

**Proposition 1.** Let $G$ be r-regular with $n$ vertices and $r \geq 2$. Then

$$DWKB(G) = \frac{(2+\sqrt{2})n(n-1)r}{2}.$$

**Proof:** Let $G$ be an $r$-regular graph with $n$ vertices and $r \geq 2$ and $\frac{nr}{2}$ edges. Then $d_{dn}(v) = n-1$ for every $v$ in $G$.

From definition,

$$DWKB(G) = \sum_{uv \in E(G)} \left( d_{dn}(u) + d_{dn}(v) + \sqrt{d_{dn}(u)^2 + d_{dn}(v)^2} \right)$$

$$= \frac{nr}{2}\left( (n-1) + (n-1) + \sqrt{(n-1)^2 + (n-1)^2} \right)$$

$$= \frac{(2+\sqrt{2})n(n-1)r}{2}.$$

**Corollary 1.1.** Let $C_n$ be a cycle with $n \geq 3$ vertices. Then

$$DWKB(C_n) = (2+\sqrt{2})n(n-1).$$

**Corollary 1.2.** Let $K_n$ be a complete graph with $n \geq 3$ vertices. Then

$$DWKB(K_n) = \frac{(2+\sqrt{2})n(n-1)^2}{2}.$$

**Proposition 2.** Let $G$ be r-regular with $n$ vertices and $r \geq 2$. Then

$${}^{m}DWKB(G) = \frac{nr}{2(2+\sqrt{2})(n-1)}.$$

**Proof:** Let $G$ be an $r$-regular graph with $n$ vertices and $r \geq 2$ and $\frac{nr}{2}$ edges. Then $d_{dn}(v) = n-1$ for every $v$ in $G$.

$${}^{m}DWKB(G) = \sum_{uv \in E(G)} \frac{1}{d_{dn}(u) + d_{dn}(v) + \sqrt{d_{dn}(u)^2 + d_{dn}(v)^2}}$$

$$= \frac{nr}{2} \frac{1}{(n-1) + (n-1) + \sqrt{(n-1)^2 + (n-1)^2}}$$

$$= \frac{nr}{2(2+\sqrt{2})(n-1)}.$$

**Corollary 2.1.** Let $C_n$ be a cycle with $n \geq 3$ vertices. Then

$${}^{m}DWKB(C_n) = \frac{n}{(2+\sqrt{2})(n-1)}.$$

**Corollary 2.2.** Let $K_n$ be a complete graph with $n \geq 3$ vertices. Then

$$^{m}DWKB(K_n) = \frac{n}{2(2+\sqrt{2})}.$$

**Proposition 3.** Let $P_n$ be a path with $n$□3 vertices. Then

$$DWKB(P_n) = 2(n-1) + (2+\sqrt{2})(n-3)(n-1).$$

**Proof:** Let $P_n$ be a path with $n$□3 vertices. Clearly, $P_n$ has two types of edges based on the uphill degree of end vertices of each edge as follows:

$E_1 = \{uv$ □ $E(P_n) \mid d_{dn}(u)=0,\ d_{dn}(v)=n-1\}$, $|E_1| = 2$.
$E_2 = \{uv$ □ $E(P_n) \mid d_{dn}(u)= d_{dn}(v)=n-1\}$, $|E_2| = n-3$.

$$DWKB(P_n) = \sum_{uv\in E(P_n)} \left(d_{dn}(u) + d_{dn}(v) + \sqrt{d_{dn}(u)^2 + d_{dn}(v)^2}\right)$$

$$= 2\left(0 + (n-1) + \sqrt{0^2 + (n-1)^2}\right) + (n-3)\left((n-1) + (n-1) + \sqrt{(n-1)^2 + (n-1)^2}\right)$$

$$= 4(n-1) + (2+\sqrt{2})(n-3)(n-1).$$

**Proposition 4.** Let $P_n$ be a path with $n$□3 vertices. Then

$$^{m}DWKB(P_n) = \frac{1}{(n-1)} + \frac{n-3}{(2+\sqrt{2})(n-1)}.$$

**Proof:** We obtain

$$^{m}DWKB(P_n) = \sum_{uv\in E(P_n)} \frac{1}{d_{dn}(u) + d_{dn}(v) + \sqrt{d_{dn}(u)^2 + d_{dn}(v)^2}}$$

$$= \frac{2}{0 + (n-1) + \sqrt{0^2 + (n-1)^2}} + \frac{(n-3)}{(n-1) + (n-1) + \sqrt{(n-1)^2 + (n-1)^2}}$$

$$= \frac{1}{(n-1)} + \frac{n-3}{(2+\sqrt{2})(n-1)}.$$

## 3. Results for Wheel Graphs

Let $W_n$ be a wheel with $n$+1vertices and $2n$ edges, $n$□4. Then there are two types of edges based on the uphill degree of end vertices of each edge as follows:

$E_1 = \{uv$ □□$E(W_n) \mid d_{dn}(u) = n,\ d_{dn}(v) = n-1\}$, $|E_1| = n$.
$E_2 = \{uv$ □□$E(W_n) \mid d_{dn}(u) = d_{dn}(v) = n-1\}$, $|E_2| = n$.

**Theorem 1.** Let $W_n$ be a wheel with $n$+1vertices and $2n$ edges, $n$□4. Then

$$DWKB(W_n) = (4+\sqrt{2})n^2.$$

**Proof.** We deduce

$$DWKB(W_n) = \sum_{uv \in E(W_n)} \left( d_{dn}(u) + d_{dn}(v) + \sqrt{d_{dn}(u)^2 + d_{dn}(v)^2} \right)$$

$$= n\left(n + (n-1) + \sqrt{n^2 + (n-1)^2}\right) + n\left((n-1) + (n-1) + \sqrt{(n-1)^2 + (n-1)^2}\right)$$

$$= n\left(2n - 1 + \sqrt{2n^2 - 2n + 1}\right) + (2 + \sqrt{2})n(n-1).$$

**Theorem 2.** Let $W_n$ be a wheel with $n$+1vertices and $2n$ edges, $n \geq 4$. Then

$$^{m}DWKB(W_n) = \frac{n}{2n - 1 + \sqrt{2n^2 - 2n + 1}} + \frac{n}{(2 + \sqrt{2})(n-1)}.$$

**Proof.** We obtain

$$^{m}DWKB(W_n) = \sum_{uv \in E(W_n)} \frac{1}{d_{dn}(u) + d_{dn}(v) + \sqrt{d_{dn}(u)^2 + d_{dn}(v)^2}}$$

$$= \frac{n}{n + (n-1) + \sqrt{n^2 + (n-1)^2}} + \frac{n}{(n-1) + (n-1) + \sqrt{(n-1)^2 + (n-1)^2}}$$

$$= \frac{n}{2n - 1 + \sqrt{2n^2 - 2n + 1}} + \frac{n}{(2 + \sqrt{2})(n-1)}.$$

## 4. Results for Gear Graphs

A bipartite wheel graph is a graph obtained from $W_n$ with $n$+1 vertices adding a vertex between each pair of adjacent rim vertices and this graph is denoted by $G_n$ and also called as a gear graph. Clearly, $|V(G_n)| = 2n+1$ and $|E(G_n)| = 3n$. A gear graph $G_n$ is depicted in Figure 1.

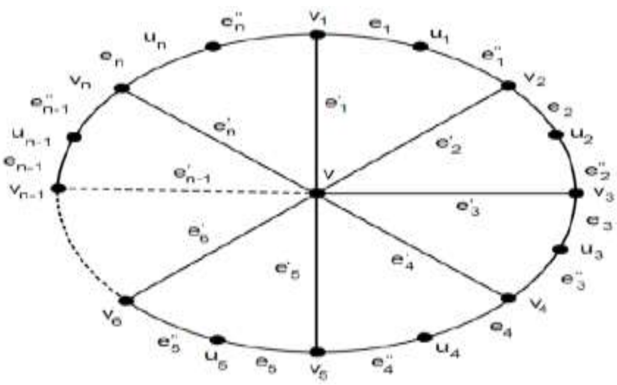

**Figure 1. Gear graph $G_n$**

Let $G_n$ be a gear graph with $3n$ edges, $n \geq 4$. Then there are two types of edges based on the uphill degree of end vertices of each edge as follows:

$E_1 = \{uv \in E(G_n) \mid d_{dn}(u) = 2n, d_{dn}(v) = 2\}$, $|E_1| = n$.
$E_2 = \{uv \in E(G_n) \mid d_{dn}(u) = 2, , d_{dn}(v) = 0\}$, $|E_2| = 2n$.

**Theorem 3.** Let $G_n$ be a gear graph with $2n$+1vertices, $n \geq 3$. Then

$$DWKB(G_n) = 10n + 2n^2 + 2n\sqrt{n^2 + 1}.$$

**Proof:** We deduce

$$DWKB(G_n) = \sum_{uv \in E(G_n)} \left( d_{dn}(u) + d_{dn}(v) + \sqrt{d_{dn}(u)^2 + d_{dn}(v)^2} \right)$$

$$= n\left(2n + 2 + \sqrt{(2n)^2 + 2^2}\right) + 2n\left(2 + 0 + \sqrt{2^2 + 0^2}\right)$$

$$= 10n + 2n^2 + 2n\sqrt{n^2 + 1}.$$

**Theorem 4.** Let $G_n$ be a gear graph with $2n+1$vertices, $n \geq 3$. Then

$$^{m}DWKB(G_n) = \frac{n}{2n + 2 + 2\sqrt{n^2 + 1}} + \frac{n}{2}.$$

**Proof:** We deduce

$$^{m}DWKB(G_n) = \sum_{uv \in E(G_n)} \frac{1}{d_{dn}(u) + d_{dn}(v) + \sqrt{d_{dn}(u)^2 + d_{dn}(v)^2}}$$

$$= \frac{n}{2n + 2 + \sqrt{(2n)^2 + 2^2}} + \frac{2n}{2 + 0 + \sqrt{2^2 + 0^2}}$$

$$= \frac{n}{2n + 2 + 2\sqrt{n^2 + 1}} + \frac{n}{2}.$$

**5. Results for Helm Graphs**

The helm graph $H_n$ is a graph obtained from $W_n$ (with $n+1$ vertices) by attaching an end edge to each rim vertex of $W_n$. Clearly, $|V(H_n)| = 2n+1$ and $|E(H_n)| = 3n$. A graph $H_n$ is shown in Figure 2.

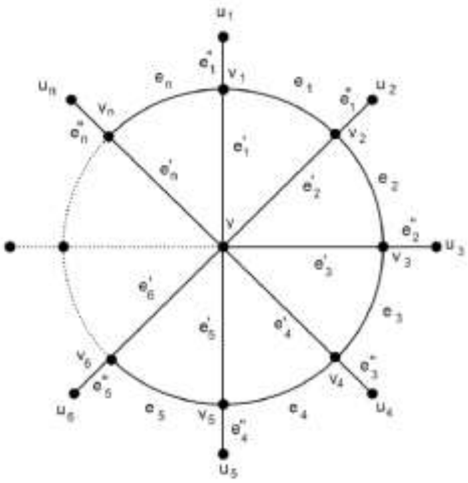

**Figure 2. Helm graph $H_n$**

Let $H_n$ be a helm graph with $3n$ edges, $n \geq 3$. Then $H_n$ has three types of the uphill degree of edges as follows:

$E_1 = \{uv \in E(H_n) \mid d_{dn}(u) = 2n,\ d_{dn}(v) = 2n - 1\}$. $\quad |E_1| = n$.

$E_2 = \{uv \in E(H_n) \mid d_{dn}(u) = d_{dn}(v) = 2n - 1\}$. $\quad |E_2| = n$.

$E_3 = \{uv \in E(H_n) \mid d_{dn}(u) = 2n - 1,\ d_{dn}(v) = 0\}$. $\quad |E_3| = n$.

**Theorem 5.** Let $H_n$ be a helm graph with $2n+1$ vertices, $n\square 3$. Then

$$DWKB(H_n) = n\left(4n-1+\sqrt{8n^2-4n+1}\right)+\left(4+\sqrt{2}\right)n(2n-1).$$

**Proof:** We obtain

$$DWKB(H_n) = \sum_{uv\in E(H_n)}\left(d_{dn}(u)+d_{dn}(v)+\sqrt{d_{dn}(u)^2+d_{dn}(v)^2}\right)$$

$$= n\left(2n+2n-1+\sqrt{(2n)^2+(2n-1)^2}\right)+n\left((2n-1)+(2n-1)+\sqrt{(2n-1)^2+(2n-1)^2}\right)$$

$$+n\left((2n-1)+0+\sqrt{(2n-1)^2+0^2}\right)$$

$$= n\left(4n-1+\sqrt{8n^2-4n+1}\right)+\left(4+\sqrt{2}\right)n(2n-1).$$

**Theorem 6.** Let $H_n$ be a helm graph with $2n+1$vertices, $n\square 3$. Then

$$^{m}DWKB(H_n) = \frac{n}{4n-1+\sqrt{8n^2-4n+1}}+\frac{n}{\left(2+\sqrt{2}\right)(2n-1)}+\frac{n}{2(2n-1)}.$$

**Proof:** We deduce

$$^{m}DWKB(H_n) = \sum_{uv\in E(H_n)}\frac{1}{d_{dn}(u)+d_{dn}(v)+\sqrt{d_{dn}(u)^2+d_{dn}(v)^2}}$$

$$= \frac{n}{2n+2n-1+\sqrt{(2n)^2+(2n-1)^2}}+\frac{n}{(2n-1)+(2n-1)+\sqrt{(2n-1)^2+(2n-1)^2}}$$

$$+\frac{n}{(2n-1)+0+\sqrt{(2n-1)^2+0^2}}$$

$$= \frac{n}{4n-1+\sqrt{8n^2-4n+1}}+\frac{n}{\left(2+\sqrt{2}\right)(2n-1)}+\frac{n}{2(2n-1)}.$$

## 6. RESULTS AND DISCUSSION: CHLOROQUINE

Chloroquine is an antiviral compound (drug) which was discovered in 1934 by H.Andersag. This drug is medication primarily used to prevent and treat malaria.

Let $G$ be the chemical structure of chloroquine. This structure has 21 vertices and 23 edges, see Figure 3.

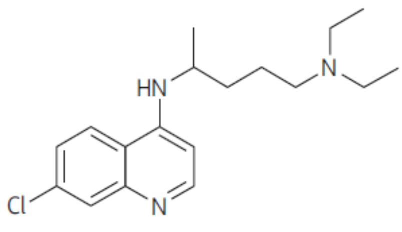


**Figure 3. Chemical structure of chloroquine**

From Figure 3, we obtain that
$\{(d_{dn}(u), d_{dn}(v))\setminus uv \in E(G)\}$ has 13 edge set partitions.

**Table 1. Edge set partitions of chloroquine**

| | | | | | |
|---|---|---|---|---|---|
| $d_{dn}(u), d_{dn}(v)\setminus uv \in E(G)$ | (9,9) | (2,9) | (1,9) | (2,7) | (1,7) |
| Number of edges | 2 | 2 | 1 | 1 | 2 |
| | (2,5) | (1,4) | (2,2) | (1,1) | (0,9) |
| | 1 | 1 | 4 | 1 | 2 |
| | (0,5) | (0,4) | (0,1) | | |
| | 2 | 2 | 2 | | |

In the following theorem, we compute the different versions of multiplicative arithmetic-geometric indices of chloroquine.

**Theorem 7.** Let $G$ be the chemical structure of chloroquine. Then

$$DWKB(G) = 199 + 27\sqrt{2} + 2\sqrt{85} + \sqrt{82} + \sqrt{53} + 2\sqrt{50} + \sqrt{29} + \sqrt{17}.$$

**Proof:** We obtain

$$DWKB(G) = \sum_{uv\in E(G)} \left(d_{dn}(u) + d_{dn}(v) + \sqrt{d_{dn}(u)^2 + d_{dn}(v)^2}\right)$$

$$= 2\left(9+9+\sqrt{9^2+9^2}\right) + 2\left(2+9+\sqrt{2^2+9^2}\right) + 1\left(1+9+\sqrt{1^2+9^2}\right) + 1\left(2+7+\sqrt{2^2+7^2}\right) + 2\left(1+7+\sqrt{1^2+7^2}\right)$$
$$+1\left(2+5+\sqrt{2^2+5^2}\right) + 1\left(1+4+\sqrt{1^2+4^2}\right) + 4\left(2+2+\sqrt{2^2+2^2}\right) + 1\left(1+1+\sqrt{1^2+1^2}\right) + 2\left(0+9+\sqrt{0^2+9^2}\right)$$
$$+2\left(0+5+\sqrt{0^2+5^2}\right) + 2\left(0+4+\sqrt{0^2+4^2}\right) + 2\left(0+1+\sqrt{0^2+1^2}\right)$$

After simplification, we obtain the desired result.

**Theorem 8.** Let $G$ be the chemical structure of chloroquine. Then

$$^{m}DWKB(G) = \frac{2}{18+9\sqrt{2}} + \frac{2}{11+\sqrt{85}} + \frac{1}{10+\sqrt{82}} + \frac{1}{9+\sqrt{53}} + \frac{2}{8+\sqrt{50}} + \frac{1}{7+\sqrt{29}} + \frac{1}{5+\sqrt{17}}$$
$$+ \frac{3}{2+\sqrt{2}} + \frac{281}{180}.$$

**Proof:** We deduce

$$^{m}DWKB(G) = \sum_{uv\in E(G)} \frac{1}{d_{dn}(u) + d_{dn}(v) + \sqrt{d_{dn}(u)^2 + d_{dn}(v)^2}}$$

$$= \frac{2}{9+9+\sqrt{9^2+9^2}} + \frac{2}{2+9+\sqrt{2^2+9^2}} + \frac{1}{1+9+\sqrt{1^2+9^2}} + \frac{1}{2+7+\sqrt{2^2+7^2}} + \frac{2}{1+7+\sqrt{1^2+7^2}}$$

$$+\frac{1}{2+5+\sqrt{2^2+5^2}}+\frac{1}{1+4+\sqrt{1^2+4^2}}+\frac{4}{2+2+\sqrt{2^2+2^2}}+\frac{1}{1+1+\sqrt{1^2+1^2}}+\frac{2}{0+9+\sqrt{0^2+9^2}}$$

$$+\frac{2}{0+5+\sqrt{0^2+5^2}}+\frac{2}{0+4+\sqrt{0^2+4^2}}+\frac{2}{0+1+\sqrt{0^2+1^2}}.$$

After simplifying, we get the desired result.

## 7. RESULTS AND DISCUSSION: HYDROXYCHLOROQUINE

Hydroxychloroquine is another antiviral compound (drug) which has antiviral activity very similar to that of chloroquine. These compounds have been repurposed for the treatment of a number of other conditions including HIV, systemic lupus erythmatosus and rheumatoid arthritis.

Let $H$ be the chemical structure of hydroxychloroquine. This structure has 22 vertices and 24 edges, see Figure 4.

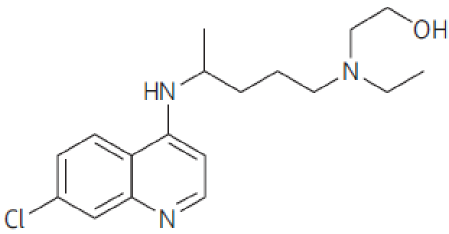


**Figure 4. Chemical structure of hydroxychloroquine**

From Figure 4, we obtain that

$\{(d_{dn}(u), d_{dn}(v))\backslash uv$ □ $E(H)\}$ has 14 edge set partition

**Table 2. Edge set partitions of hydroxychloroquine**

| | | | | | |
|---|---|---|---|---|---|
| $d_{dn}(u), d_{dn}(v) \backslash uv$ □ $E(H)$ | (9,9) | (2,9) | (1,9) | (2,8) | (1,8) |
| Number of edges | 2 | 2 | 1 | 2 | 1 |
| | (2,5) | (1,4) | (2,2) | (1,1) | (0,9) |
| | 1 | 1 | 5 | 1 | 2 |
| | (0,5) | (0,4) | (0,2) | (0,1) | |
| | 2 | 2 | 1 | 1 | |

**Theorem 9.** Let $H$ be the chemical structure of hydroxychloroquine. Then

$$DWKB(H)=209+29\sqrt{2}+2\sqrt{85}+\sqrt{82}+5\sqrt{17}+\sqrt{65}+\sqrt{29}.$$

**Proof:** We obtain

$$DWKB(H)=\sum_{uv\in E(H)}\left(d_{dn}(u)+d_{dn}(v)+\sqrt{d_{dn}(u)^2+d_{dn}(v)^2}\right)$$

$$= 2\left(9+9+\sqrt{9^2+9^2}\right)+2\left(2+9+\sqrt{2^2+9^2}\right)+1\left(1+9+\sqrt{1^2+9^2}\right)+2\left(2+8+\sqrt{2^2+8^2}\right)+1\left(1+8+\sqrt{1^2+8^2}\right)$$
$$+1\left(2+5+\sqrt{2^2+5^2}\right)+1\left(1+4+\sqrt{1^2+4^2}\right)+5\left(2+2+\sqrt{2^2+2^2}\right)+1\left(1+1+\sqrt{1^2+1^2}\right)+2\left(0+9+\sqrt{0^2+9^2}\right)$$
$$+2\left(0+5+\sqrt{0^2+5^2}\right)+2\left(0+4+\sqrt{0^2+4^2}\right)+1\left(0+2+\sqrt{0^2+2^2}\right)+1\left(0+1+\sqrt{0^2+1^2}\right)$$

Simplification gives the desired result.

**Theorem 10.** Let $H$ be the chemical structure of hydroxychloroquine. Then

$$^{m}DWKB(H)=\frac{2}{18+9\sqrt{2}}+\frac{2}{11+\sqrt{85}}+\frac{1}{10+\sqrt{82}}+\frac{2}{5+\sqrt{17}}+\frac{1}{9+\sqrt{65}}+\frac{1}{7+\sqrt{29}}$$
$$+\frac{5}{4+2\sqrt{2}}+\frac{1}{2+\sqrt{2}}+\frac{59}{45}.$$

**Proof:** We obtain

$$^{m}DWKB(H)=\sum_{uv\in E(H)}\frac{1}{d_{dn}(u)+d_{dn}(v)+\sqrt{d_{dn}(u)^2+d_{dn}(v)^2}}$$
$$=\frac{2}{9+9+\sqrt{9^2+9^2}}+\frac{2}{2+9+\sqrt{2^2+9^2}}+\frac{1}{1+9+\sqrt{1^2+9^2}}+\frac{2}{2+8+\sqrt{2^2+8^2}}+\frac{1}{1+8+\sqrt{1^2+8^2}}$$
$$+\frac{1}{2+5+\sqrt{2^2+5^2}}+\frac{1}{1+4+\sqrt{1^2+4^2}}+\frac{5}{2+2+\sqrt{2^2+2^2}}+\frac{1}{1+1+\sqrt{1^2+1^2}}+\frac{2}{0+9+\sqrt{0^2+9^2}}$$
$$+\frac{2}{0+5+\sqrt{0^2+5^2}}+\frac{2}{0+4+\sqrt{0^2+4^2}}+\frac{1}{0+2+\sqrt{0^2+2^2}}+\frac{1}{0+1+\sqrt{0^2+1^2}}.$$

After simplification, we get the required result.

## 8. RESULTS AND DISCUSSION: REMDESIVIR

Remdesivir is an antiviral drug which was developed by the biopharmaceutical company Gilead Sciences. Let $R$ be the molecular graph of remdesivir. This graph has 41 vertices and 44 edges.

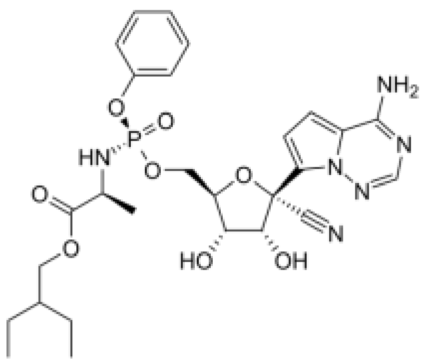


**Figure 5. Chemical structure of remdesivir**

From Figure 5, we obtain that

$\{(d_{dn}(u),d_{dn}(v)) \setminus uv \in E(R)\}$ has 20 edge set partitions.

**Table 3. Edge set partitions of remdesivir**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $d_{dn}(u),d_{dn}(v)$ | (9,19) | (7,19) | (9,9) | (2,9) | (1,9) | (7,7) | (1,7) | (6,6) |
| $\setminus uv \in E(R)$ | 1 | 1 | 3 | 2 | 2 | 2 | 1 | 1 |
| Number of edges | (4,6) | (1,6) | (1,5) | (4,4) | (2,2) | (1,1) | (0,19) | (0,9) |
| | 2 | 4 | 1 | 4 | 2 | 3 | 2 | 1 |
| | (0,7) | (0,6) | (0,5) | (0,1) | | | | |
| | 3 | 4 | 3 | 2 | | | | |

**Theorem 11.** Let $R$ be the chemical structure of remdesivir. Then

$$DWKB(R)=516+70\sqrt{2}+\sqrt{442}+\sqrt{410}+2\sqrt{85}+2\sqrt{82}+\sqrt{50}+4\sqrt{13}+4\sqrt{57}+\sqrt{26}.$$

**Proof:** We obtain

$$DWKB(R)=\sum_{uv\in E(R)}\left(d_{dn}(u)+d_{dn}(v)+\sqrt{d_{dn}(u)^2+d_{dn}(v)^2}\right)$$

$$=1\left(9+19+\sqrt{9^2+19^2}\right)+1\left(7+19+\sqrt{7^2+19^2}\right)+3\left(9+9+\sqrt{9^2+9^2}\right)+2\left(2+9+\sqrt{2^2+9^2}\right)+2\left(1+9+\sqrt{1^2+9^2}\right)$$
$$+2\left(7+7+\sqrt{7^2+7^2}\right)+1\left(1+7+\sqrt{1^2+7^2}\right)+1\left(6+6+\sqrt{6^2+6^2}\right)+2\left(4+6+\sqrt{4^2+6^2}\right)+4\left(1+6+\sqrt{1^2+6^2}\right)$$
$$+1\left(1+5+\sqrt{1^2+5^2}\right)+4\left(4+4+\sqrt{4^2+4^2}\right)+2\left(2+2+\sqrt{2^2+2^2}\right)+3\left(1+1+\sqrt{1^2+1^2}\right)+2\left(0+19+\sqrt{0^2+19^2}\right)$$
$$+1\left(0+9+\sqrt{0^2+9^2}\right)+3\left(0+7+\sqrt{0^2+7^2}\right)+4\left(0+6+\sqrt{0^2+6^2}\right)+3\left(0+5+\sqrt{0^2+5^2}\right)+2\left(0+1+\sqrt{0^2+1^2}\right).$$

After simplifying the above equation, we get the desired result.

**Theorem 12.** Let $R$ be the chemical structure of hydroxychloroquine. Then

$$^{m}DWKB(R)=\frac{1}{28+\sqrt{442}}+\frac{1}{26+\sqrt{410}}+\frac{1}{6+3\sqrt{2}}+\frac{2}{11+\sqrt{85}}+\frac{2}{10+\sqrt{82}}$$
$$+\frac{2}{14+7\sqrt{2}}+\frac{1}{8+\sqrt{50}}+\frac{1}{12+6\sqrt{2}}+\frac{1}{5+\sqrt{13}}+\frac{4}{7+\sqrt{37}}$$
$$+\frac{1}{6+\sqrt{26}}+\frac{5}{2+\sqrt{2}}+\frac{1}{19}+\frac{1}{18}+\frac{3}{14}+\frac{1}{3}+\frac{3}{10}+1.$$

**Proof:** We obtain

$$^{m}DWKB(R)=\sum_{uv\in E(R)}\frac{1}{d_{dn}(u)+d_{dn}(v)+\sqrt{d_{dn}(u)^2+d_{dn}(v)^2}}$$

$$= \frac{1}{9+19+\sqrt{9^2+19^2}} + \frac{1}{7+19+\sqrt{7^2+19^2}} + \frac{3}{9+9+\sqrt{9^2+9^2}} + \frac{2}{2+9+\sqrt{2^2+9^2}} + \frac{2}{1+9+\sqrt{1^2+9^2}}$$

$$+ \frac{2}{7+7+\sqrt{7^2+7^2}} + \frac{1}{1+7+\sqrt{1^2+7^2}} + \frac{1}{6+6+\sqrt{6^2+6^2}} + \frac{2}{4+6+\sqrt{4^2+6^2}} + \frac{4}{1+6+\sqrt{1^2+6^2}}$$

$$+ \frac{1}{1+5+\sqrt{1^2+5^2}} + \frac{4}{4+4+\sqrt{4^2+4^2}} + \frac{2}{2+2+\sqrt{2^2+2^2}} + \frac{3}{1+1+\sqrt{1^2+1^2}} + \frac{2}{0+19+\sqrt{0^2+19^2}}$$

$$+ \frac{1}{0+9+\sqrt{0^2+9^2}} + \frac{3}{0+7+\sqrt{0^2+7^2}} + \frac{4}{0+6+\sqrt{0^2+6^2}} + \frac{3}{0+5+\sqrt{0^2+5^2}} + \frac{2}{0+1+\sqrt{0^2+1^2}}.$$

By simplifying, the result is obtained.

## 9. Conclusion

In this paper, the downhill Kepler Banhatti and modified downhill Kepler Banhatti indices of a graph are defined. Also these newly defined the downhill Kepler Banhatti indices of some standard graphs, wheel graphs, gear graphs, helm graphs and some chemical structures are determined.

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