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Modelling and Analysis of a Far-Field Wireless Power Transfer Efficiency Equation using CST Microwave Studio

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Abstract

The Friis transmission equation is a fundamental principle of radio engineering. It is a method for studying radio wave propagation. It helps to calculate the power received by an antenna considering both the distances between its antennas and the characteristics of both antennas. To comprehend and improve WPT systems, it is essential to utilize this equation, particularly for long-range scenarios. Using the CST Microwave simulation software, this paper co nducts a study to verify the validity of the Friis transmission formula for far-field radio wave analysis.

Keywords: Modelling, WPT, Friis, transmission, Antenna.

1. INTRODUCTION

In radio communication theory, the Friis transmission equation is a formula for determining antenna power in optimal free-space conditions. Nevertheless, some people have raised concerns about its accuracy in practical settings, such as near-field environments or in intricate materials [2]. This study investigates the current techniques for verifying outcomes through full-wave electromagnetic simulations. The near-field region is the part of the electromagnetic spectrum that lies within half a wavelength from the antenna [3]. Radiative electromagnetic radiation is emitted by the antenna in the far-field region, and it can travel endlessly through free space. The antenna's radiation resistance is a crucial factor in the efficient transmission of power to this field [3].

2. BACKGROUND

If the antennas are separated by $R > 2\ (D^2/\ \lambda)$, then the Friis transmission formula indicates that the transmitted and received power will be correlated, with D being the maximum physical dimension of the larger antenna and being its operating wavelength [3]. Initially, assume that the transmitting antenna displays isotropic radiation characteristics, with equal radiation in every direction. E.g. The Friis transmission equation requires parameters such as: (1) Antenna Gain (Gt and Gr), which measures the concentration of radiated power in a particular direction by an antenna. Greater gain results in better transmission or reception efficiency, while wavelength (λ) is inversely proportional to operating frequency. Shorter wavelengths are typically associated with higher frequencies and wider bandwidth for signal propagation patterns. The longer wavelengths at lower frequencies can be advantageous for long-range communication. The Friis equation is a useful tool for computing the theoretical power transfer efficiency between antennas. The evaluation of a WPT system's feasibility and limitations requires this. The factors that affect received power can be utilized to optimize antenna parameters and distance, ultimately resulting in efficient power transfer. However, the details are still unclear.

3. PROBLEM STATEMENT

The question of transmission efficiency of wireless power transfer (WPT) in the far-field suffers on three fronts, (i) DC-to-RF; (ii) Free space path loss (FSPL) in which the power decays according to the square of the distance between antennas and (iii) the RF-to-DC efficiency [2,3]. The challenge of WPT is on increasing the efficiency transmission in the free space. Several scholars have proposed the use of use of special antenna substrates to enhance directivity [4]. The Friis transmission equation governs the propagation of radio frequency (RF) in the far-field. However, there is limited study of the Friis transmission formula using the software modelling tools to estimate the end efficiency of designed antennas.

4. LITERATURE REVIEW

The Friis formula was derived by [1,2,7] as shown in eqn. (11). The equation is used to solve for the power received from one antenna of gain G_1 when transmitting from another antenna of gain G_2 separated by a distance R, and operating at wavelength lambda λ. [20, 21] discussed using MATLAB and HFSS together to simulate phased array antennas. MATLAB'S Phased Array Toolbox permits for the defining of array geometry and calculating array factors.

5. FAR-FIELD PROPAGATION

In the far-field zone of antennas, where electromagnetic wave-fronts are planar, the Friis transmission equation is most accurate and can provide valid plane wave estimates [1, 2, 4]. This is particularly true for RF measurements. The Friis transmission formula relies on the idealized assumption of free-space conditions, while neglecting environmental factors like atmospheric attenuation, multipath effects, and surface reflections. Despite the development of multiple adaptations, its reliability is most firmly established in the antenna far-field region [2].

6. RADIATION PATTERN

A radiation pattern is a visual representation of the antenna's radioactivity and characteristics as reflected by space coordinates, typically located in the far-field region, where factors like radiation intensity, electromagnetic field strength, phase behavior, and polarization are scrutinized [5].

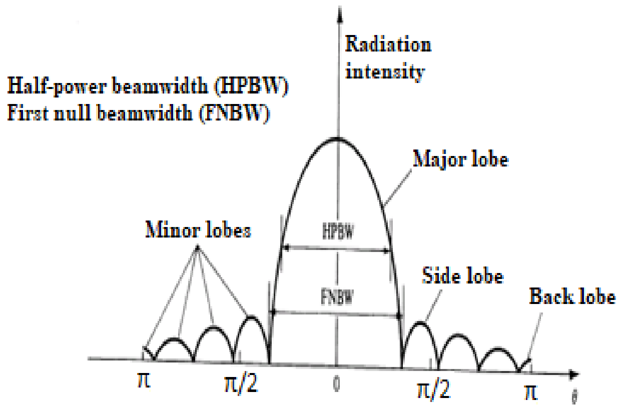

Figure 1: Antenna Radiation Pattern in Cartesian coordinate system Showing Various Types of Lobes [5].

From Figure 1, it can be seen that the radiation intensity of the antenna is nothing but the power per unit solid angle radiated in the direction of the antenna [5].

7. TOTAL RADIATED POWER AND RADIATION EFFICIENCY

An antenna serves as a transducer that converts guided electrical energy, delivered from the transmitter's output circuitry, into radiated electromagnetic fields propagating through free space. The antenna radiation efficiency, denoted η_r, characterizes the proficiency of this energy conversion and is mathematically defined as the ratio of radiated power P_{rad} to the net input power P_{in} provided to the antenna terminals [6]. In reception mode, the antenna acts as a transducer that converts incident electromagnetic radiation into electrical signals. The efficiency of this energy conversion process is equivalent to the antenna's radiation efficiency, assuming reciprocal behavior as governed by antenna theory [1, 6]. Antenna radiation efficiency is taken into account when antenna gain is calculated. Therefore, many antenna manufacturers choose to provide only antenna gain information, which is sufficient for most applications [4]. The overall power radiated by the antenna is obtained by performing a surface integral of the power density function $Pd(r,\theta,\phi)$ across a closed spherical boundary encompassing the radiating structure (eqt. 1, 2) [6].

$$P_{rad} = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} P_d\left(r,\theta,\phi\right) r^2 \sin\theta d\phi d\theta \ldots\ldots. (1)$$

$$P_{rad} = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} P_d\left(\theta,\phi\right) \sin\theta d\phi d\theta \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

The radiated power Prad from the antenna is inherently limited to be less than or equal to the input power Pin delivered to its terminals. A fraction of Pin, denoted as Ploss, is expended as thermal energy within the antenna's conductive and dielectric materials due to internal dissipation (eqt. 3-18) [6].

$$P_{rad} = P_{in} - P_{loss} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$$P_r = P_{in}\left(1 - \frac{P_{loss}}{P_{in}}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

$$\eta_r = 1 - \frac{P_{loss}}{P_{in}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

The quantity ηr, referred to as radiation efficiency, characterizes how effectively an antenna radiates input power. The power lost (P_{loss}) comprises multiple components: dielectric loss (P_{ld}), conductor loss (P_{lc}), and in some cases, surface wave loss in the substrate material (P_{sw}). Each of these loss mechanisms is associated with its own efficiency—ηd for dielectric losses, ηc for conductor losses, and ηsw for surface wave losses. If the individual losses are small, the total radiation efficiency can be approximated as the product of individual parts:

$$\eta_r = 1 - \frac{P_{ld} + P_{lc} + P_{sw}}{P_{in}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

$$\eta_r \approx \left(1 - \frac{P_{ld}}{P_{in}}\right)\left(1 - \frac{P_{lc}}{P_{in}}\right)\left(1 - \frac{P_{sw}}{P_{in}}\right) = \eta_c \eta_d \eta_{sw} \ldots. (7)$$

8. FREE SPACE PROPAGATION MODEL

The free space propagation model characterizes the transmission of radio frequency (RF) energy through an ideal medium—typically vacuum or unobstructed air—assuming a direct line-of-sight (LOS) path between the transmitting and receiving antennas. It neglects environmental impairments such as reflection, diffraction, or scattering, thereby serving as a baseline model for evaluating signal attenuation and path loss under idealized conditions. This conceptual framework is pivotal for grasping the fundamental principles of electromagnetic wave propagation and underpins the development of more advanced channel models. Transmission efficiency, in this context, is quantified by the ratio of received power to transmitted power, as articulated through the Friis transmission equation, which incorporates antenna directivities and operating wavelength. (eqt. 9) [6]. The Friis transmission equation was developed to estimate the received power based on the transmitted power, factoring in the operating wavelength, the distance between transmitting and receiving antennas, and the respective antenna gains [7, 16, 17, 18]. The transmission efficiency is also known as beam efficiency.

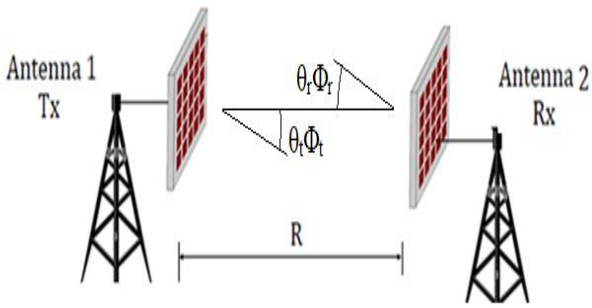

Figure 2: Line of Sight Link

Friis equation shall be accurate if distance between transmitter and receiver is more than minimum distance (R_{min}) given by equation (8)

$$\frac{P_r}{P_t} = \left(\frac{\lambda}{4\pi R}\right)^2 D_t(\theta_t,\phi_t) D_r(\theta_r,\phi_r) \qquad (8)$$

Where, D_r is receiving antenna directivity, G_t is transmitting antenna directivity, R is propagation distance between antennas in meters, λ is the frequency wavelength in meters, P_r is radiating power in Watts, P_t is transmitting power in Watts.

The minimum transmission distance R_{min} is given by the expression of (9);

$$R_{\min} \geq \frac{2D^2}{\lambda} \qquad (9)$$

Where D is the largest linear dimension of an antenna, meters. λ is wavelength corresponding to operating frequency. The formula for calculating radiating power density (P_D) at any propagation distance 'R' is given by the equation (10);

$$P_D = \frac{4P_{in}\eta_t}{\pi D_b^2} \qquad (10)$$

Where; D_b is the diameter of beam at distance 'R', meters, P_{in} is the input power in Watts and η_t is reflector efficiency. If the power density at the propagation distance 'R' is to be calculated, the microwave beam diameter b_D must be known at that distance. Assume that the microwave beam has propagated to the distance beyond Fresnel region then the beam angle in degrees [7] is given by the equation (12)

In order to find out the beam diameter at any propagating distance, it is assumed that the beam is divergent from the transmitter itself. The spherical coordinate (r, θ, φ) system is used to specify the field strength within the spherical surface. However, the radiation pattern is independent of the radius of the sphere where r >> λ. A three-dimensional radiation pattern ensures the radiation for all angles of θ and φ in Figure 2 [8].

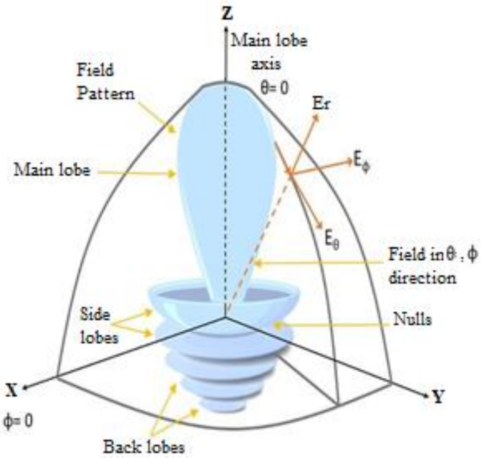

Figure 3: 3D Wave Propagation [8]

The received power as a function of directivity is given by the equation (14);

$$P_r = \left(\frac{\lambda}{4\pi R}\right)^2 \eta_t D_t \eta_r D_t P_t \text{ .... (11)}$$

The beam angle θ can be approximately calculated using the equation given by (14)

$$\theta = \tan^{-1}\left(\frac{\lambda(b_D - D)}{4D^2}\right) \text{ .... (12)}$$

The beam diameter b_D at any propagating distance in the Fresnel region [9] can be calculated using the equation given by (13).

$$b_D = D\sqrt{1+\left(\frac{20d}{D}\right)^2} \text{ .... (13)}$$

If the effective area of receiving antenna is known, the received power on receiver can be calculated by multiplying effective area with power density calculated using equation (16). The received power is given by equation (14) [6];

$$P_r = P_D\left(\frac{\pi D_r^2}{4}\right) = \frac{4P_{in}\eta_t}{\pi D_b^2}\left(\frac{\pi D_r^2}{4}\right) = p_{in}\eta_t\left(\frac{D_r^2}{D_b^2}\right) \text{ .... (14)}$$

Where; Dr is the receiver diameter in meters and η_t is transmitter efficiency. The formula for WPT system efficiency is given by equations (15) and (16).

$$\eta = \frac{P_r}{P_{in}} = \left\{ \frac{p_{in}\eta_t \left(\frac{D_r^2}{D_b^2} \right)}{P_{in}} \right\} = \eta_t \left(\frac{D_r^2}{D_b^2} \right) \quad \text{if } D_r < D_b \ldots\ldots\ldots (15)$$

$$\eta = \frac{P_r}{P_{in}} = \left\{ \frac{p_{in}\eta_t \left(\frac{D_r^2}{D_b^2} \right)}{P_{in}} \right\} = \eta_t \left(\frac{D_r^2}{D_b^2} \right) = \eta_t \quad \text{if } D_r \geq D_b. \ldots\ldots (16)$$

9. MODELING IN CST MICROWAVE STUDIO

CST Microwave Studio (CST MWS), a specialized module within the CST Studio Suite, is a high-fidelity computational tool designed for three-dimensional electromagnetic field simulation. It enables the accurate modeling, analysis, and optimization of high-frequency components such as antennas, filters, and microwave circuits. CST MWS leverages a range of numerical solvers tailored for distinct frequency bands and physical configurations, thereby supporting comprehensive virtual prototyping across RF, microwave, and millimeter-wave domains. Its integration within the broader CST Studio Suite facilitates multi-physics co-simulation and interoperability with other electromagnetic analysis tools, making it a valuable asset in both academic research and industrial design workflows. [10]. Antenna pattern measurements are typically conducted in the far-field region, where the spatial distribution of radiated power becomes effectively invariant with respect to distance. The parameters that pertain to microstrip patch antenna design are governed by the resonant frequency, f_0; antenna material permittivity, Ɛr; and the material thickness, h [11]

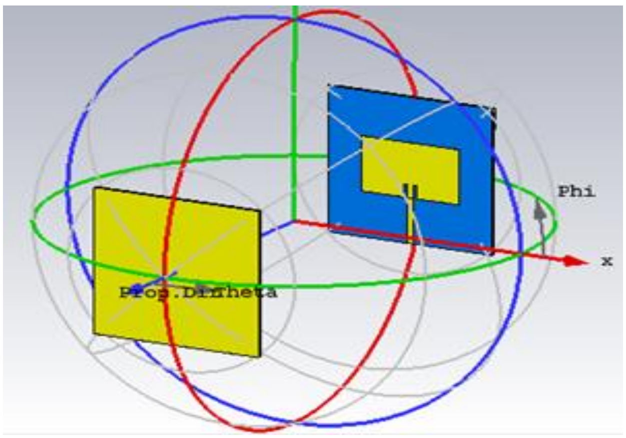

Figure 4: Far-Field Simulation of FSPL of Transmitter Antenna to Receiver Antenna

This arises from the requirement that the antenna's emitted signal be capable of propagating over extended distances; consequently, the spatial distribution of radiated power must be analyzed at electrically large separations to ensure accurate pattern characterization.. From the Friis formula, it's well understood that Pr can be maximized by minimizing the path loss. Path loss becomes more pronounced with higher operating frequencies and greater separation between the transmitter and receiver antennas. This behavior is illustrated in Figure 5, where both components are represented within the CST Microwave Studio simulation environment.. Note that the transmitter is a microstrip patch antenna, with the Lumped Port 1 voltage excitation [11]. The Tx and Rx are a square patch antenna which is insert fed as shown in Figure 4. The substrate is a dielectric with a permittivity equal to 4.08, and that L=W=80 mm, so that the patch is to resonate at 4.5 GHz. The simulation in CST software was done with the following parameters; (1) The resonance frequency was selected as 2.4GHz with a range of 2-2.8GHz; (2) the patch antenna dimension (L and W) is calculated by using microstrip transmission line formula [11, 12]: (3) The dielectric is calculated with permittivity of 4.08.

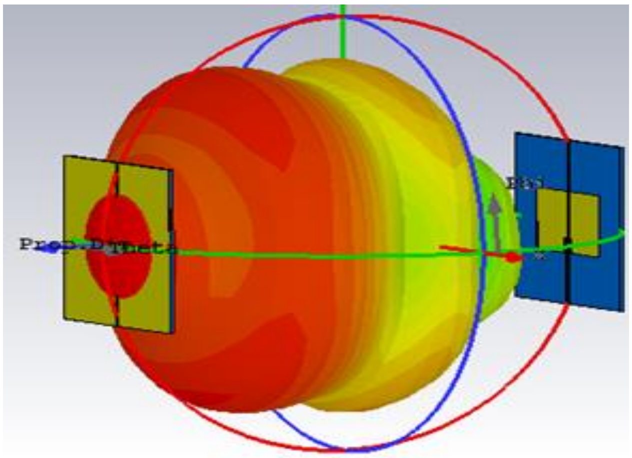

Figure 5: Simulated Far-field Pattern Visualization in CST Studio at 2.4 GHz

10. RESULTS

In CST Studio, S-parameters (or scattering parameters) are numerical representations of how microwave and RF signals behave when interacting with a circuit or structure. [13] They describe the power transfer and reflection characteristics of a network at different frequencies, making them crucial for understanding and optimizing high-frequency designs. Figure 6 shows the resulting reflection coefficient (S11) as a function of frequency [14]. It is noticed that the reflection coefficient reaches a minimum value of –14.65 dB at 2.4 GHz with a bandwidth ranging from 2 to 2.8 GHz. In addition, Figure 7 shows the resulting 2D radiation pattern in the two planes E (φ = 0◦) and H (φ = 90◦) for the proposed antenna. Figure 5 shows the radiation efficiency of -2.764 dB with a directivity of 6.792 dBi.

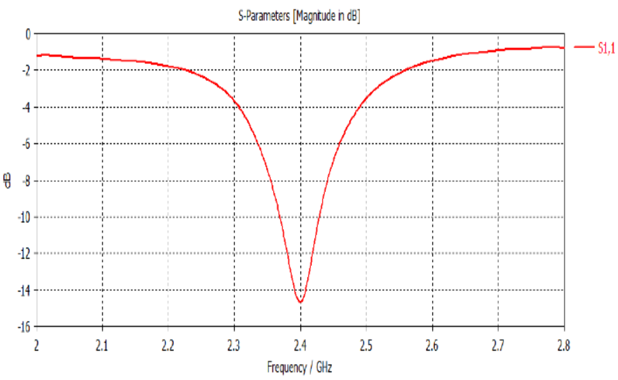

Figure 6: Reflection Coefficient S_{11} Parameter

The Smith chart serves as a graphical tool for representing the conversion between the reflection coefficient and the corresponding input impedance. [15].

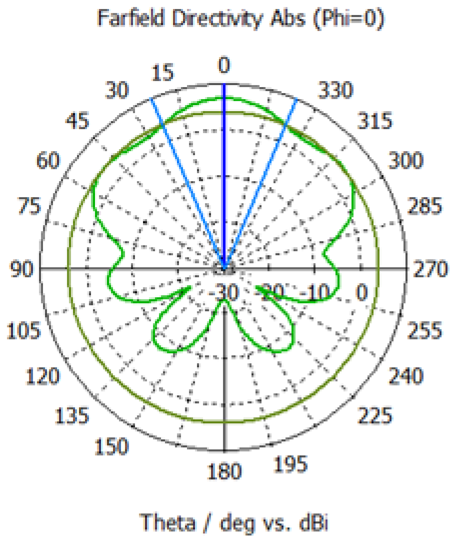

Figure 7: Radiation Pattern in Polar Coordinates

11. CONCLUSION

The primary aim of this paper was to lay the groundwork for understanding the Friis transmission equation, serving as an introductory step toward improving antenna directivity through the use of metamaterials. Overall, the Friis transmission equation proves to be an essential analytical framework for evaluating and refining wireless power transfer, especially in scenarios involving long-distance, free-space propagation.

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