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Exploring the Dynamic Interdependencies of Exchange Rates, Exports, and Imports in Kazakhstan: Johansen Cointegration & A Vector Autoregression Analysis

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Abstract

This study examines the relationships among exchange rate, exports, and imports in the Kazakhstan economy using monthly data from November 1993 to October 2022. All variables are non-stationary at levels but become stationary after first differencing, indicating they are integrated of order one, I(1). Granger Causality Analysis suggested that a unidirectional causal relationship from exchange rate to imports and a bidirectional relationship between exports and imports, highlighting their interdependence. Johansen Cointegration Analysis confirmed that the existence of stable long-run relationships among the variables at levels. A VAR(1,9) model further supports these findings, with significant coefficients in the exchange rate equation indicating that lags 3 and 4 of the exchange rate and lag 9 of imports drive depreciation, while lags 6 and 9 of exports promote appreciation through foreign currency inflows. The import equation shows mean-reversion at lag 1 and a linkage with export lags 1 and 8, reflecting export-driven import demand. The export equation indicates mean-reversion at lags 1, 2, 7, 8, and 9 with import lags 4, 5, 8, and 9 boosting exports, likely due to imported inputs for oil production. The VAR model is stable, with all inverse roots inside the unit circle, ensuring reliable dynamics.

Keywords: Stationarity Test, ADF Test, Johansen Cointegration Test, VAR Stability Condition Check, Vector Autoregressive Model, Variance Covariance Matrix, Inverse Roots of AR Characteristics Polynomial.

JEL Classifications: F1, F4, F13, M210

1. Introduction:

Kazakhstan's economic history from 1990 to 2022 is a story of transformation from a Soviet Republic to an independent, resource-dependent market economy. Gaining independence in 1991 amid the collapse of the Soviet Union, Kazakhstan faced hyperinflation, structural reforms, and a shift to capitalism. Its economy became heavily reliant on oil and gas, which accounted for about 60–70% of exports by the



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2010s, making it vulnerable to global commodity prices, geopolitical events, and external shocks. The exchange rate (primarily the Kazakhstani tenge, KZT, against the US dollar, USD), exports, and imports evolved in tandem, influenced by oil booms, financial crises, devaluations, and policy shifts toward diversification. Kazakhstan's economy in the early 1990s was marked by the dissolution of Soviet supply chains, hyperinflation, and privatization. As part of the USSR until 1991, it relied on centralized planning, with exports focused on raw materials like oil, metals, and grains to other Soviet republics. Independence triggered a severe recession: GDP fell by 13% in 1994 alone, and inflation peaked at 1,880% in 1994.

Kazakhstan initially used the Soviet ruble, but hyperinflation led to the introduction of the tenge (KZT) on November 15, 1993, at an initial rate of about 4.7 KZT/USD. The tenge depreciated rapidly due to fiscal deficits, low reserves, and the ruble's collapse. By 1994, it devalued to around 50 KZT/USD amid a currency crisis. Further devaluations followed: to 60 KZT/USD in 1995 and 88 KZT/USD by 1999, influenced by the 1998 Russian financial crisis (Russia was a key trade partner).

Exports were low and volatile, starting from ~\$3–5 billion in the early 1990s (mostly intra-Soviet trade in oil, metals, and agriculture). By 1995, total exports were about \$5.2 billion, rising modestly to \$5.9 billion in 1998 before dipping to \$5.6 billion in 1999 due to the Russian crisis. Top exports: Crude oil (emerging as key post-Tengiz deal with Chevron in 1993), ferrous metals, copper, and wheat. Main partners: Russia (40–50% share), Ukraine, and China. The Tengiz oil field discovery (proven reserves ~25 billion barrels) began attracting foreign investment, but production ramp-up was slow. Imports mirrored exports at low levels, around \$4–6 billion annually, focused on machinery, vehicles, and consumer goods from Russia and Europe. In 1995, imports were ~\$3.8 billion, rising to \$4.3 billion in 1998 and falling to \$3.7 billion in 1999. Trade balance was often positive but small (~\$1–2 billion surplus), as the economy contracted.

2. Literature Review:

Matesanz Gómez and Fugarolas Álvarez-Ude (2006) analysed the impact of the real exchange rate (RER) on Argentina's trade balance (TB) from 1962 using VAR-based cointegration tests and impulse response functions. Their findings indicate that the Marshall-Lerner condition holds under fixed exchange rate regimes but not under flexible ones, despite periods of RER overvaluation. Short-run TB dynamics rarely exhibit the J-curve pattern, except before 1991. The 2002 devaluation, following the abandonment of the currency board, likely improved the TB, suggesting that flexible exchange rate policies may support sustainable economic growth in Argentina.

Bhattarai, K. B. & Armah, M. K. (2005) they examined the impact of real exchange rates on Ghana's trade balance using 1970–2000 data. Deriving real exchange rates from preferences and technology, they apply a small price-taking economy model. Cointegration analyses confirmed stable long-run relationships between exports, imports, and real exchange rates. Short-run devaluation effects are contractionary per Marshall-Lerner-Robinson conditions, but long-run elasticities approach unity. The study recommends coordinating exchange rate and demand management policies based on long-run economic fundamentals for an improved trade balance.

Duasa (2008) examines the impact of exchange rate shocks on Malaysia's import and export prices using a vector error correction model (VECM) with monthly data from January 1999 to December 2006. Through variance decompositions and impulse response functions, the study identified the significant



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effects of exchange rate shocks on import prices, though the pass-through was incomplete. The results underscore dynamic interactions among nominal exchange rates, money supply, and trade prices, revealing the complex transmission of exchange rate changes to Malaysia's trade sector.

Objectives of our Study:

- I. **Investigate the Stationarity of Time Series Data**: To determine whether the time series data for exchange rate, imports, and exports in the Kazakhstan economy are stationary or not by using appropriate statistical tests (e.g., Augmented Dickey-Fuller Test).
- II. **Identify Short-Run and Long-Run Relationships**: Assess whether short-run and/or long-run relationships exist among exchange rate, imports, and exports in Kazakhstan, using cointegration tests (e.g., Johansen Cointegration Test) for long-run relationships and Granger causality tests for short-run dynamics.
- III. **Examine the Dynamic Relationships Among Variables**: Analyse the dynamic interrelationships among exchange rate, imports, and exports in the Kazakhstan economy using a Vector Autoregression (VAR) model depending on the stationarity and cointegration properties of the data.
- IV. To evaluate the stability of long-run cointegrating relationships among exchange rate, exports, and imports in the Kazakhstan economy and verify the stability of the associated VAR model by examining the roots of the characteristic polynomial A(L).

Data Source: I collected the monthly secondary datasets for the period from 1993.XI to 2022:X from the International Monetary Fund (IMF) – International Financial Statistics (IFS) and various issues of the Direction of Trade Statistics Yearbook.

Test of Stationarity: Augmented Dickey-Fuller (ADF) Unit Root Test

Stationarity of Exchange rate (E_t), Export(X) and Import(M) series have been studied through the Augmented Dickey Fuller (ADF) tests. The basic ADF equation estimated with appropriate changes under different assumptions are

$$\begin{split} \Delta E_t &= \alpha_1 + \beta_1 t + \gamma_1 E_{t-1} + \delta_{1i} \sum_{i=1}^n \Delta E_{t-1} + \epsilon_{1i}......(1) \\ \Delta X_t &= \alpha_2 + \beta_2 t + \gamma_2 X_{t-1} + \delta_{2i} \sum_{i=1}^n \Delta X_{t-1} + \epsilon_{2i}.....(2) \\ \Delta M_t &= \alpha_3 + \beta_3 t + \gamma_3 M_{t-1} + \delta_{3i} \sum_{i=1}^n \Delta M_{t-1} + \epsilon_{3i}.....(3) \\ \text{where } \epsilon_{1t} &\sim \text{iidN}(0, \sigma_{\epsilon_1}^2), \ \epsilon_{2t} &\sim \text{iidN}(0, \sigma_{\epsilon_2}^2) \ \text{and} \ \epsilon_{3t} &\sim \text{iidN}(0, \sigma_{\epsilon_3}^2), \\ \text{and } \Delta E_t &= (E_t - E_{t-1}), \ \Delta X_t = (X_t - X_{t-1}) \ \text{and} \ \Delta M_t = (M_t - M_{t-1}) \end{split}$$

The optimal lag (k) may be determined through Akaike Information Criterion, Schwartz Information Criterion, Hannan-Quinn Information criterion etc.

The test assesses whether past values of one variable provide statistically significant information about the future values of another, beyond the information contained in the variable's own past. Below the Granger Causality Test evaluates the null hypothesis that lagged values of one variable (e.g., exchange rate) do not improve the prediction of another variable (e.g., export) when the latter's own lagged values



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are already included in the model. It is based on an F-test comparing a restricted model (excluding the lagged variable) to an unrestricted model (including it).

Table:1
Results of the Augmented Dickey Fuller (Unit Root Test)

(Automatic based on SIC, Maxlag=16) [Sample: - 1993: XI -2022:X]

		, <u>-</u>		
Covertex	Variable	ADF Test	Prob*	Remarks
Country	variable	Stat.	Value	Kemarks
	E_t	0.676	0.991	Non-Stationary
	ΔE_t	-7.550	0.000	Stationary
Kazakhstan	X_t	-0.433	0.900	Non-Stationary
Kuzukiistaii	ΔX_t	-4.246	0.001	Stationary
	M_t	-1.078	0.725	Non-Stationary
	ΔM_t	-22.766	0.000	Stationary

where E_t , X_t and M_t stands for Exchange Rate, Export and Import at level and ΔE_t , ΔX_t and ΔM_t stands for 1st difference of Exchange Rate, Export and Import of Kazakhstan. It is observed from the Table: 1 that

- i. Exchange rate(E_t), export(X) and import(M) series at level are having unit roots even at 10% level of significance.
- ii. the first difference of Exchange Rate, export and import are free from unit roots even at 1% level of significance.
- iii. Exchange Rate (E_t) , $Export_t$ and $Import_t$ are non-stationary and I(1) variable in the economy of Kazakhstan.
- iv. Exchange Rate (ΔE_t) , $\Delta Export_t$ and $\Delta Import_t$ are stationary i.e all are I(0) variables.
- v. Exchange Rate (E_t) , $Export_t$ and $Import_t$ series contain 'unit roots' and, therefore, these series are 'non-stationary' by nature.

Johansen Cointegration Test: Both the Johansen (1988) and the Stock and Watson(1988) methodologies rely heavily on the relationship between the rank of the matrix and the characteristic roots. The Johansen cointegration test equation is presented below:

$$\Delta y_t = \gamma + \pi y_{t-1} + \sum_{i=1}^{\rho-1} \pi_i \Delta y_{t-i} + \mu_t \dots (4)$$

where, γ is the vector of constants, y_t is the m dimensional vector of variables, (i.e., e_t , p_t in our analysis), ρ is the number of lags, μ_t is the error vector, which is multivariate normal and independent across observations.

$$\pi = -(1 - \sum_{i=1}^{\rho} A_i)$$
 and $\pi = -\sum_{i=i+1}^{\rho} A_i$(5)

Here, the rank of the matrix π is equal to the number of independent cointegrating vectors.

The estimated equations for the Bi-Variate Granger Causality Test:

Unrestricted Model (Includes all Lag):



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$$Exchange\ Rate_t = \alpha_0 + \sum_{i=1}^9 \alpha_{1i} Exchange\ Rate_{t-i} + \sum_{i=1}^9 \beta_{1i} Export_{t-i} + \epsilon_{1t}.....(6)$$

$$Export_{t} = \gamma_{0} + \sum_{i=1}^{9} \gamma_{1i} Export_{t-i} + \sum_{i=1}^{9} \delta_{1i} Exchange \ Rate_{t-i} + \epsilon_{2t} \dots (7)$$

 α_{1i} , β_{1i} , γ_{1i} , δ_{1i} , σ_{1i} and ρ_{1i} : Coefficient of lagged Exchange Rate, Export and Import ϵ_{1t} , ϵ_{2t} and ϵ_{3t} : All are Error term.

Test: Compare the residual sum of squares (RSS) of restricted vs. unrestricted models using an F-statistic:

$$F = \frac{(RSS_r - \frac{RSS_u}{p})}{RSS_u/(n-2p-1)}.$$
(9)

 $RSS_r = Restricted RSS$

 $RSS_u = Unrestricted RSS$

P = Number of lags (9)

n = Number of observations (338)

if F> F critical (or p-value < 0.05), reject the null, indicating Granger Causality.

Table: 2
Grager Causality Test for Exchange rate, Import & Export

rest for Enchange rate, import at Emp			
Pairwise Granger Causality Tests			
Sample: 1993:M11 – 2022:M10			
Lags: 9			
			P-
Null Hypothesis:	Obs	F-Stat.	Value
ΔExport does not Granger Cause ΔEx-			
Rate	338	1.508	0.144
ΔEx-Rate does not Granger Cause			
ΔExport	338	1.273	0.250
ΔImport does not Granger Cause ΔEx-			
Rate	338	0.965	0.469
ΔEx-Rate does not Granger Cause			
ΔImport	338	2.144	0.026
ΔImport does not Granger Cause ΔExport	t338	3.540	0.000
ΔExport does not Granger Cause ΔImport	338	3.155	0.001

In the above Granger Causality Analysis, we investigated that the F-stat. value 2.144 is significant with p-value 0.026 at 5% level of significance i.e. reject the null hypothesis which implies that the exchange rate does not Granger Cause import. Similarly, F-stat. value 3.540 and 3.155 with p-value 0.000 and 0.001 also significance which indicated that the Δ Import does not Granger Cause Δ Export and Δ Export does not Granger Cause Δ Import. There was no causal relationship between exchange rate and export. Again, we see that there was unidirectional causal relationship between the variables (Exchange rate and Import) and bi-directional Causal relation between the variables Export and Import in Kazakhstan economy.



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Johansen Cointegration Test:

The test uses two complementary statistics to estimate r:

1. Trace Test:

Trace Stat. =
$$-T \sum_{i=r+1}^{k} ln(1-\hat{\lambda}_i)$$
....(10)

Where T=Number of observation (348)

 $\hat{\lambda}_i = E$ stimated eigenvalues of Π

k = Number of Variables(3)

Test $H_0 = r$ cointegrating relationships vs. $H_1: r + 1$ or more.

2. Max-Eigen Value Test:

$$Max - Eigen = -T In(1 - \hat{\lambda}_{r+1}).....(11)$$

Focus on the Max-Eigen values to Test H_0 : r vs. H_1 : r+1

Hypothesis Testing: Reject H_0 if the statistic exceeds the critical value at the 0.05 level or the p-value (MacKinnon-Haug-Michelis, 1999) is less than 0.05. The process starts with r = 0 and proceeds until H_0 is not rejected.

Table :3
Results of the Johansen Cointegration Tests for Exchange rate,
Import and Export at Level Period: 1993:M11 - 2022:M10

Trend Assumption: No Deterministic Trend

Lag Interval in first difference: 1 4

I Unrestricted	I Unrestricted Cointegration Rank λ _{trace} Test						
Variables Invo	olved: Exchang	ge rate, Ir	nport and Exp	oort at Level			
Null	Alternative	Eigen	Trace Stat.	Test	P-		
Hypothesis	Hypothesis	Value	(λ_{trace})	Critical	Valu		
				Values	e		
	(5%)						
r≤0	r=0	0.078	42.803	24.276	0.000		
r≤1	r=1	0.034	14.900	12.321	0.018		
r≤2	r=2	0.009	3.004	4.130	0.098		

II Unrestric	II Unrestricted Cointegration Rank λ _{max} Test						
Variables Inv	volved: Exch	ange rate,	Import and Ex	port at Level			
Null	Alternativ	Eigen	Max. Eigen	Test	P-		
Hypothesis	e	Value	Stat. (λ_{max})	Critical	Value		
	Hypothesi	Hypothesi Values					
	s (5%)						
r≤0	r=0	0.078	27.903	17.797	0.001		
r≤1	r=1	0.034	11.896	11.225	0.038		
r≤2	r=2	0.009	3.004	4.130	0.098		



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Finding From the Table- 3

Results of the Johansen Cointegration Test, as given in the Table 3, show that in the period 1993:M11 - 2022:M10.

- i. for r = 0 against r > 1, $\lambda_{trace}(0,0) = 42.803$ exceeds the corresponding critical value at 5% level. This implies that the null hypothesis of the 'absence of cointegration' (r = 0) between exchange rate and export at level has been rejected at 5% level.
- ii. for $r \le 1$ against r = 1, $\lambda_{trace}(1,1) = 14.900$ also exceeds the corresponding critical value even at 5% level. This implies that the 'null hypothesis of not more than 'one cointegrating relation' is accepted even at 5% level.
- iii. for r \leq 2 against r = 2, λ_{trach} (2,2) = 3.004 falls short of the corresponding critical value even at 5% level. This implies that the 'null hypothesis of not more than 'one cointegrating relation' is accepted even at 5% level.
- iv. for r \leq 0 against r = 1, λ_{max} (0,0) = 27.903 exceeds—the corresponding critical value even at 5% level. This implies that the 'null hypothesis of not more than 'one cointegrating relation' is accepted even at 5% level.
- v. for r \leq 1 against r = 1, $\lambda_{trace}(1,1)$ = 11.896 also exceeds the corresponding critical value even at 5% level. This implies that the 'null hypothesis of not more than 'one cointegrating relation' is accepted even at 5% level.
- vi. for $r \le 2$ against r = 2, $\lambda_{trach}(2,2) = 3.004$ falls short of the corresponding critical value even at 5% level. This implies that the 'null hypothesis of not more than 'one cointegrating relation' is accepted even at 5% level.

From the above table we see that the Max-Eigen value test indicates 2 cointegrating equations. Both tests agree on 2 cointegrating relationships, confirming the presence of two stable long-run equilibria. Exchange rate is loosely tied to export, while Import is strongly proportional to Export.

Overview of the findings from the Johansen Cointegration Test

(for the period 1993:M11 - 2022:M10.)

It is observed for Period 1993:M11 - 2022:M10.

- i. Exchange rate, export and import are CI(1,0).
- ii. There exists a cointegrating relation between exchange rate, export and import at level.

Economic Implications of the Findings of Cointegration Study:

The existence of cointegration between exchange rate and export, import at level implies that there did exist a long run relationship between exchange rate of currencies concerned with the export and import prevailing over the period 1993:M11 - 2022:M10.

Unrestricted Cointegrating Coefficients of Johansen Cointegration Test:

The unrestricted cointegrating equations are linear combinations of the form:

$$\beta_1' y_t = b_{11} Exchange \ rate_t + b_{12} Import_t + b_{13} Export_t = 0....(12)$$

$$\beta_2'y_t = b_{21}Exchange\ rate_t + b_{22}Import_t + b_{23}Export_t = 0....(13)$$

Where
$$\beta_1' = [b_{11}, b_{12}, b_{13}]$$
 and $\beta_2' = [b_{21}, b_{22}, b_{23}]$ are the row of the β matrix.



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Estimated model from the equations (12) and (13)

$$0.008Exchange\ rate_t + 0.002Import_t - 0.003Export_t = 0....(14)$$

$$0.004Exchange\ rate_t - 0.001Import_t + 0.001Export_t = 0....(15)$$

These equations suggest linear combinations that are stationary, but the coefficients are small and unnormalized, making interpretation challenging without scaling.

Normalized Cointegrating coefficients in Johansen Cointegration Test:

Exchange rate
$$-0.220Import + 0.360Export = 0.....(16)$$

S.E 0.040 0.055

Here we see that the both S.E value are significant in the normalized cointegrating equations.

Cointegrating Equations:

Vector 1:

$$Exchange\ rate - 0.021Export = 0.....(17)$$

S.E. 0.017

Vector 2:

$$Import - 1.542Export = 0.....(18)$$

S.E 0.087

In vector 1 we see that the coefficient value 0.021 with S.E value is insignificant. Similarly, in Vector 2 the value of S.E is significant.

The Vector Autoregressive (VAR) Model

The Vector Autoregressive (VAR) Model for Exchange Rate(E_t), Import(M_t) and Export(X_t) is as follows.

$$E_{t} = \sum_{i=1}^{n} \beta_{1i} E_{t-i} + \sum_{i=1}^{n} \gamma_{1i} M_{t-i} + \sum_{i=1}^{n} \delta_{1i} X_{t-i} + \varepsilon_{1t} \dots (19)$$

$$M_{t} = \sum_{i=1}^{n} \beta_{2i} E_{t-i} + \sum_{i=1}^{n} \gamma_{2i} M_{t-i} + \sum_{i=1}^{n} \delta_{2i} X_{t-i} + \varepsilon_{2t} \dots (20)$$

$$X_{t} = \sum_{i=1}^{n} \beta_{3i} E_{t-i} + \sum_{i=1}^{n} \gamma_{3i} M_{t-i} + \sum_{i=1}^{n} \delta_{3i} X_{t-i} + \varepsilon_{3t} \dots (21)$$

Here $E_t = \Delta Exchange\ rate_t$, $M_t = \Delta Import_t\ and\ X_t = \Delta Export_t$ represent the first differenced stationary time series dataset for Exchange rate, $Import_t\ and\ Export_t$ respectively over period 1994M09 2022M10. Since $E_t \sim I(1)$, $M_t \sim I(1)\ and\ X_t \sim I(1)$ the stationarity of Exchange rate, $Import_t\ and\ Export_t$ is ensured through the first difference filtering of Exchange Rate, $Import_t\ and\ Export_t$ respectively.

The equations (19),(20) and (21) represent 'Seemingly Unrelated Regression Equations' (SURE) since the joint estimation of these equations considers and uses the 'Contemporaneous Var-Covariance matrix (Ω) of the cross-equation error terms involved such that $\Omega = \text{Var-Covar}(\mathbf{u}_{1t}, \mathbf{u}_{2t})$ where Ω is a Positive Definite Matrix.

Structure of the Variance-Covariance Matrix

For k = 3 variables $\{X_1, X_2, X_3\}$, the variance-covariance matrix is given by:



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$$\sum = \begin{bmatrix} Var(X_1) & Cov(X_1, X_2) & Cov(X_1, X_3) \\ Cov(X_2, X_1) & Var(X_2) & Cov(X_2, X_3) \\ Cov(X_3, X_1) & Cov(X_3, X_2) & Var(X_3) \end{bmatrix} = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_2^2 & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_3^2 \end{bmatrix}$$

Diagonal elements (σ_i^2) represent the variance of each variable. Off-diagonal elements (σ_{ij}) represent the covariance between pairs of variables. The matrix is symmetric because $Cov(X_i, X_i) = Cov(X_i, X_i)$

If the variables are standardized (zero mean and unit variance), the covariance matrix becomes the correlation matrix, where off-diagonal elements are correlation coefficients.

$$\sum = \begin{bmatrix} 71.638 & -163.656 & -331.280 \\ -163.656 & 162069.453 & 51342.804 \\ -331.280 & 52342.804 & 86183.761 \end{bmatrix}$$

This matrix corresponds to a VAR model with three variables: Exchange Rate, Imports, and Exports.

From the above matrix the diagonal elements represent the variances of the residuals for each variable, indicating the variability of the shocks (innovations) that are not explained by the VAR model's lagged terms. The residual variance of the exchange rate is relatively small. If the exchange rate is measured per USD, this suggests moderate volatility in the exchange rate residuals after accounting for its own lags and the lags of imports and exports. Similarly, he residual variance for imports is very large, indicating high variability in import shocks. This could reflect Kazakhstan's reliance on imported goods (e.g., machinery, consumer goods), which may be subject to large, unpredictable fluctuations due to global supply chain disruptions, changes in domestic demand, or currency fluctuations.

On the other hand, the residual variance for exports is also large but smaller than imports. Kazakhstan's exports are dominated by oil and gas, so this variance captures shocks like changes in global commodity prices or production disruptions. The off-diagonal elements suggested that the covariances between the residuals of different variables, showing how shocks to one variable are contemporaneously related to shocks to another. The covariance value between Exchange Rate and Imports was -163.656 which means that the negative covariance indicates that a positive shock to the exchange rate (e.g., depreciation of the Kazakhstani tenge, KZT, meaning more KZT per USD) is associated with a negative shock to imports. In economic terms, a weaker tenge makes imports more expensive, reducing import demand, which aligns with this negative relationship. For Kazakhstan, where imports include capital goods and consumer products, a depreciation could significantly curb import volumes.

Similarly, the covariance value for Exchange Rate and Exports was -331.280 which means that the negative covariance suggests that a positive shock to the exchange rate (depreciation) is associated with a negative shock to exports. This is counterintuitive, as depreciation typically boosts exports by making them cheaper in foreign markets. The covariance value for the Imports and Exports was 51342.804 indicated that the positive covariance indicates that positive shocks to imports are associated with positive shocks to exports. This reflects Kazakhstan's trade dynamics, where higher imports (e.g., machinery for oil production) may coincide with higher exports (e.g., increased oil output). This positive relationship is



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typical in resource-dependent economies, where imports of capital goods support export-oriented industries.

Selection of Lag Length in the VAR Estimation

The optimum lag length (m) has been determined on the basis of some Information Criteria, like Akaike Information Criterion (AIC), Schwartz Information Criterion (SIC), Hannan-Quin Information Criterion (HQIC), Sequential Modified LR Test Statistic (SMLST), Forecast Prediction Error(FPE) Statistic etc. The Table 8.1 presents the relevant lag length statistics as given by these criteria.

Table: 4
VAR Lag Order Selection Criterion

VAR La	VAR Lag Order Selection Criteria							
Endoge	Endogenous variables: Exchange Rate, Import,							
Export								
Exogen	ous variabl	es: C						
Sample:	1993M11	2022M10						
Included	d observati	ons: 338						
Lag	LogL	LR	FPE	AIC	SC	HQ		
0	-6145.042	NA	1.26e+12	36.379	36.413*	36.392		
1	-6120.472	48.558	1.15e+12	36.287	36.422	36.341*		
2	-6105.876	28.588	1.12e+12	36.254	36.491	36.348		
3	-6094.522	22.036	1.10e+12	36.240	36.579	36.375		
4	-6077.415	32.898	1.05e+12	36.192	36.633	36.368		
5	-6070.720	12.756	1.06e+12	36.205	36.748	36.423		
6	-6059.770	20.668	1.05e+12	36.194	36.839	36.451		
7	-6050.211	17.875	1.05e+12	36.191	36.937	36.488		
8	-6039.520	19.801	1.04e+12	36.180	37.029	36.519		
				36.156				
9	-6026.310	24.230*	*	37.106	36.534			
* indicates lag order selected by the criterion								
LR: sequential modified LR test statistic (each test at 5% level)								
	inal predict							
AIC: Akaike information criterion								
SC: Scl	SC: Schwarz information criterion							
HQ: Ha	annan-Quir	ın informa	tion criterion	n				

AIC, FPE and HQ statistics suggest for lag 9 as the optimum lag. The trial-and-error estimations, as suggested by Enders, also confirm lag 9 as the optimum lag. So, in the VAR model, consisting of equations (19), (20) and (21), the optimum lag (m) is set to be 9.

Estimated equations of the VAR(1,9) model for the Equations (19), (20) and (21)

$$E_t = 0.087E_{t-1} - 0.024E_{t-2} - 0.159E_{t-3} - 0.264E_{t-4} - 0.042E_{t-5} - 0.075E_{t-6} + 0.055E_{t-7}$$

t-stat. 1.537 -0.414 -2.750 4.372 -0.663 -1.190 0.850



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S.E	0.057	0.057	0.057	0.060	0.063	0.063	0.065
-0.055 <i>I</i> -0.7 0.0	⁷ 91 -	0.537	0.561	-0.589		$+ 0.000 M_{t-4} + 0.075 \\ 0.001$	$0.001M_{t-5} \\ 0.576 \\ 0.001$
0.0		1.455		2.131 *		$ \begin{array}{rrr} -0.003X_{t-2} - \\ 8 & -1.588 \\ 0.002 \end{array} $	$0.001X_{t-3} \\ -0.259 \\ 0.002$
-0.00 (22)	$1X_{t-4}-0.0$	$001X_{t-5} - 0$	$0.005X_{t-6}$ -	$+ 0.001X_{t-7}$	$-0.003X_{t-8}$	$-0.004X_{t-9}$ +	ϵ_{1t}
-0.369 0.002			2. 477 * 0.002	0.702 0.002	-1.757 0.002	-2.324* 0.002	
R ² =	= 0.153,	-		- stat. = 2.0 89, SIC	_	relihood = 118	86.891,
$M_t = $ t-stat S.E	-1.410	-2.321	1.2	215 1.			325
-1.146	-0.351	0.76	3 -5	.004 -	$0.027M_{t-2} - 0.430$ 0.065		0.405
+0.043 <i>l</i> 0.634 0.069	- 0.8	19 -0.	913	$0.16M_{t-8} + 0.238$ 0.069	0.584		$0.062X_{t-2} -0.683 \\ 0.091$
ϵ_{2t} (2.0.605)	3) -0.645	-1.060	-0.71		99 2.27	$3X_{t-8} + 0.069$ 7^* 0.791 9 0.087	$X_{t-9} +$
$R^2 =$	= 0.188,	=		stat.= 2.66 13, SIC	_	elihood = -24	92.274,
t-stat.	-1.580	1.133	-1.068	-1.883	0.76	$E_{-5} + 2.190E_{t-6}$ 7 0.100 5 2.191	-0.692
0.443	-0.5	43 0.1	17	0.214	1.779	3.724* 0.049	4.009*
+0.063 1.240						$-0.270X_{t-2}$ -4.780	



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$$0.051 \quad 0.050 \quad 0.051 \quad 0.048 \quad 0.062 \quad 0.066 \quad 0.066$$

$$-0.305X_{t-4} - 0.304X_{t-5} - 0.301X_{t-6} - 0.269X_{t-7} - 0.169X_{t-8} - 0.157X_{t-9} +$$

$$\epsilon_{3t} - 0.060 \quad 0.066 \quad 0.066 \quad 0.066 \quad 0.065 \quad 0.063$$

$$R^2 = 0.212, \quad Adj \ R^2 = 0.144, F - stat. = 3.097, \ Log \ Likelihood = -2385.543,$$

$$R^2 = 0.212$$
, $Adj R^2 = 0.144$, $F - stat. = 3.097$, $Log Likelihood = -2385.543$, $AIC = 14.281$, $SIC = 14.598$

Essential Features of the VAR Model

The VAR Model consisting of equations (19), (20) and (21) requires that

- exchange rate ,export and import be 'Stationary'.
- ii. the model be 'Stable'.
- iii. ϵ_{1t} , ϵ_{2t} and ϵ_{3t} be white noise terms such that $\epsilon_{1t} \sim (0, \sigma_{\epsilon_1}^2)$

$$\epsilon_{1t} \sim (0, \sigma_{\epsilon_2}^2)$$

$$\epsilon_{1t} \sim (0, \rho_{\epsilon_2}^2)$$

In this model E_t , X_t and M_t are 'Stationary' since

 Δ Exchange rate = E_t , Δ Export = X_t and Δ Import = M_t

where $Exchange\ rate_t \sim I(1)$, $Export_t \sim I(1)$ and $Import_t \sim I(1)$

Therefore $E_t = I(0)$, $X_t = I(0)$ and $M_t = I(0)$

Findings from the VAR Model (equations 22,23,24)

In the estimated equation (22,23 and 24)

- $|\acute{\beta}_{13}|<1 \text{ , } |\acute{\beta}_{14}|<1 \text{ , } |\acute{\gamma}_{19}|<1 \text{ , } |\acute{\delta}_{16}|<1 \text{ , } |\acute{\delta}_{19}|<1 \text{ , } |\acute{\gamma}_{21}|<1 \text{ , } |\acute{\delta}_{21}|<1 \text{ , }$ $|\delta_{28}| < 1$, $|\dot{\gamma}_{34}| < 1$, $|\dot{\gamma}_{35}| < 1$, $|\dot{\gamma}_{38}| < 1$, $|\dot{\gamma}_{39}| < 1$, $|\delta_{31}| < 1$, $|\delta_{32}| < 1$, $|\delta_{33}| < 1, |\delta_{34}| < 1, |\delta_{35}| < 1, |\delta_{36}| < 1, |\delta_{37}| < 1, |\delta_{38}| < 1$ and $|\delta_{39}| < 1$
- So the autoregressive and distributed lag structures are consistent.
- β_{11} , β_{12} , β_{15} , β_{16} , β_{17} , β_{18} , β_{19} , δ_{11} , δ_{12} , δ_{13} , δ_{14} , δ_{15} , δ_{17} , δ_{18} are not significant even at 10% levels.
- $\sum_{i=1}^{8} \gamma_{1i}$ insignificant even at 10% level.
- v. $\sum_{i=2}^{9} \gamma_{2i}$ insignificant even at 10% level.
- $\sum_{i=1}^{9} \beta_{3i}$ insignificant even at 10% level.
- vii. $\dot{\gamma}_{31}$, $\dot{\gamma}_{32}$, $\dot{\gamma}_{33}$, $\dot{\gamma}_{36}$, $\dot{\gamma}_{37}$, $\dot{\beta}_{18}$ and $\dot{\beta}_{19}$ insignificant even at 10% level.
- viii. $\sum_{i=2}^{9} \gamma_{2i}$ insignificant even at 10% level.
- ix. β_{13} , β_{14} , γ_{19} δ_{16} , δ_{19} are significant at 1% level.
- $\dot{\gamma}_{21}$, $\dot{\delta}_{21}$, $\dot{\delta}_{28}$ are significant at 1% level.



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- xi. $\dot{\gamma}_{34}$, $\dot{\gamma}_{35}$, $\dot{\gamma}_{38}$, $\dot{\gamma}_{39}$ are significant at 1% level
- xii. $\sum_{i=1}^{9} \delta_{3i}$ are significant even at 1% level.

Economic Interpretations of equations (22), (23) and (24)

The economic significance of the findings is as follows:

- a. Insignificant value of β_{11} , to β_{19} indicate that variations in current exchange rate were not related to those in last exchange rates. It again implies that variations in exchange rate did not exhibit a sustenance over the forthcoming period.
- b. $\hat{\beta}_{13}$, $\hat{\beta}_{14}$, $\hat{\gamma}_{19}$, $\hat{\delta}_{16}$, $\hat{\delta}_{19}$, $\hat{\gamma}_{21}$, $\hat{\delta}_{21}$, $\hat{\delta}_{28}$, $\hat{\gamma}_{34}$, $\hat{\gamma}_{35}$, $\hat{\gamma}_{38}$, $\hat{\gamma}_{39}$ being significant, even in the presence of E_{t-1} in the vector of regressors in the VAR equation for E_t , indicates that export and import 'Granger Caused' exchange rate over the period of study.

Again, the consistence of the VAR Model requires that the model be stable. The conditions of 'stability' are derived below and then we proceed to examine if these conditions are met by the estimated VAR model. Once the 'stability' conditions are satisfied, then we would examine if ϵ_{1t} , ϵ_{2t} and ϵ_{2t} are white noise by nature.

Findings from the equation (22) we examined that the coefficient is significant with t-stat. value at 1% level of significant for the own lag 3 and 4 of exchange rate which means that the 1-unit increase in the exchange rate change 3 months ago i.e. there is a depreciation in the Kazakhstan economy. Again. t-stat. value is highly significant for the lag 9 of import in the same equation indicated that a 1-unit increase in import changes 9 month ago suggested that there is a depreciation of exchange rate. Similarly, t-stat. value for lag of 6 and 9 of export is significant at 1% level explained that the due to the increase in export reduces the exchange rate change means there is appreciation and higher exports increases foreign currency inflows and reinforcing that export growth stabilizes the exchange rate.

Similarly, according to the equation (23) we noticed that t-stat. value is significant for the own lag 1 of import indicated that the 1-unit increase in import changes last month reduces current import changes, indicating mean-reversion in import growth, possibly due to budget constraints. Again, for the first and eight lags of export the t-value is also significant at 1% level. So we see that there is reflecting Kazakhstan's linkage between export revenues and import demand at the same time sustained export growth supports imports.

Findings from the estimated equation (24) we see that the coefficients for the own lags 1,2,7,8 and 9 of export are significant with t-stat. values means reinforcing mean-reversion in exports. Again, the t-stat. values for lags 4,5,8 and 9 of import are also significant and a 1-unit increase in import changes up to 9 months ago increases exports, reflecting the need for imported inputs to boost oil production.

Table: 5

VAR Stability Condition Check [Roots of the AR Characteristic Polynomial A(L)]

Endogenous Variable: Exchange rate, Import, Export Exogenous Variable: C Lag Specification: 1 9



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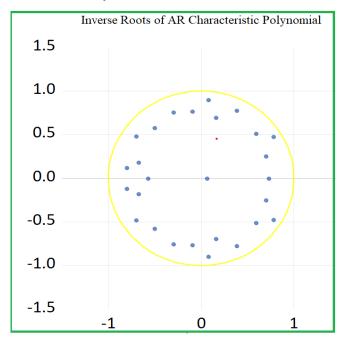
Endogenous variables: Exchange rate Import Export Exogenous variables: C Lag specification: 1 9 Root	Roots of Characteristic P	olvnomial
Import Export Exogenous variables: C Lag specification: 1 9 Root Modulus 0.9151315027981 0.781500 + 0.476155i 14 0.9151315027981 0.781500 - 0.476155i 14 0.9023322701562 0.076280 + 0.899102i 379 0.9023322701562 379 0.8645277744797 0.382051 + 0.775529i 506 0.8645277744797 0.382051 - 0.775529i 506 0.8476860053224 0.698190 - 0.480732i 0.698190 + 0.480732i 0.8142302777684 0.299274 + 0.757236i 895 0.8142302777684 0.299274 - 0.757236i 895 0.8074012496840 0.798620 + 0.118756i 946 0.8074012496840 0.798620 - 0.118756i 946 0.7827062968769 0.591660 + 0.512414i 492 0.7827062968769 0.591660 - 0.512414i 492 0.7705696647074 249 0.7626309295262 635 63		J
Exogenous variables: C Lag specification: 1 9 Root		Entitle Tare
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		Lag specification:
$\begin{array}{c} 0.9151315027981 \\ 0.781500 + 0.476155i \\ 14 \\ 0.9151315027981 \\ 0.781500 - 0.476155i \\ 14 \\ 0.9023322701562 \\ 0.076280 + 0.899102i \\ 0.076280 - 0.899102i \\ 0.382051 + 0.775529i \\ 0.382051 - 0.775529i \\ 0.8645277744797 \\ 0.382051 - 0.775529i \\ 0.698190 - 0.480732i \\ 0.698190 + 0.480732i \\ 0.698190 + 0.480732i \\ 0.0698190 + 0.480732i \\ 0.08142302777684 \\ 0.0299274 + 0.757236i \\ 0.8142302777684 \\ 0.0299274 - 0.757236i \\ 0.8074012496840 \\ 0.0798620 + 0.118756i \\ 0.8074012496840 \\ 0.07827062968769 \\ 0.591660 + 0.512414i \\ 0.7827062968769 \\ 0.591660 - 0.512414i \\ 0.7705696647074 \\ -0.094599 + 0.764741i \\ -0.094599 + 0.764741i \\ 249 \\ 0.7705696647074 \\ -0.094599 + 0.764741i \\ 249 \\ 0.7705696647074 \\ 249 \\ 0.7705696647074 \\ 249 \\ 0.7705696647074 \\ 249 \\ 0.7705696647074 \\ 249 \\ 0.7705696647074 \\ 249 \\ 0.7705696647074 \\ 249 \\ 0.7705696647074 \\ 249 \\ 0.7705696647074 \\ 249 \\ 0.7705696647074 \\ 249 \\ 0.7705696647074 \\ 249 \\ 0.7705696647074 \\ 249 \\ 0.7705696647074 \\ 249 \\ 0.7705696647074 \\ 249 \\ 0.7705696647074 \\ 249 \\ 0.7626309295262 \\ 635 \\ \end{array}$		Lag specification.
$\begin{array}{c} 0.781500 + 0.476155i & 14 \\ 0.9151315027981 \\ 0.781500 - 0.476155i & 14 \\ 0.9023322701562 \\ 0.076280 + 0.899102i & 379 \\ 0.9023322701562 \\ 0.076280 - 0.899102i & 379 \\ 0.8645277744797 \\ 0.382051 + 0.775529i & 506 \\ 0.8476860053224 \\ -0.698190 - 0.480732i & 0.8476860053224 \\ -0.698190 + 0.480732i & 0.8476860053224 \\ -0.698190 + 0.480732i & 0.8142302777684 \\ -0.299274 + 0.757236i & 895 \\ 0.8142302777684 \\ -0.299274 - 0.757236i & 895 \\ 0.8074012496840 \\ -0.798620 + 0.118756i & 946 \\ 0.8074012496840 \\ -0.798620 - 0.118756i & 946 \\ 0.7827062968769 \\ 0.591660 + 0.512414i & 492 \\ 0.7827062968769 \\ 0.591660 - 0.512414i & 492 \\ 0.7705696647074 \\ -0.094599 - 0.764741i & 249 \\ -0.094599 + 0.764741i & 249 \\ -0.501071 + 0.574920i & 635 \\ \end{array}$	Root	Modulus
$\begin{array}{c} 0.9151315027981 \\ 0.781500 - 0.476155i \\ 14 \\ 0.9023322701562 \\ 379 \\ 0.9023322701562 \\ 379 \\ 0.9023322701562 \\ 379 \\ 0.8645277744797 \\ 0.382051 + 0.775529i \\ 0.8645277744797 \\ 0.382051 - 0.775529i \\ 0.8476860053224 \\ -0.698190 - 0.480732i \\ 0.698190 + 0.480732i \\ 0.698190 + 0.480732i \\ 0.8142302777684 \\ -0.299274 + 0.757236i \\ 0.8142302777684 \\ -0.299274 - 0.757236i \\ 0.8074012496840 \\ -0.798620 + 0.118756i \\ 0.8074012496840 \\ -0.798620 - 0.118756i \\ 0.7827062968769 \\ 0.591660 - 0.512414i \\ 492 \\ 0.7705696647074 \\ -0.094599 + 0.764741i \\ 249 \\ 0.7705696647074 \\ 249 \\ 0.7705696647074 \\ 249 \\ 0.7705696647074 \\ 249 \\ 0.7705696647074 \\ 249 \\ 0.7705696647074 \\ 249 \\ 0.7705696647074 \\ 249 \\ 0.7705696647074 \\ 249 \\ 0.7705696647074 \\ 249 \\ 0.7705696647074 \\ 249 \\ 0.7705696647074 \\ 249 \\ 0.7626309295262 \\ 635 \\ \end{array}$		0.9151315027981
$\begin{array}{c} 0.781500 - 0.476155i & 14 \\ \hline 0.9023322701562 \\ 0.076280 + 0.899102i & 379 \\ \hline 0.9023322701562 \\ 0.076280 - 0.899102i & 379 \\ \hline 0.382051 + 0.775529i & 506 \\ \hline 0.8645277744797 \\ 0.382051 - 0.775529i & 506 \\ \hline 0.8476860053224 \\ -0.698190 - 0.480732i & 076 \\ \hline 0.8476860053224 \\ -0.698190 + 0.480732i & 076 \\ \hline 0.8142302777684 \\ -0.299274 + 0.757236i & 895 \\ \hline 0.8074012496840 \\ -0.798620 + 0.118756i & 946 \\ \hline 0.798620 - 0.118756i & 946 \\ \hline 0.798620 - 0.118756i & 946 \\ \hline 0.591660 + 0.512414i & 492 \\ \hline 0.591660 - 0.512414i & 492 \\ \hline 0.7705696647074 \\ -0.094599 + 0.764741i & 249 \\ \hline -0.790501071 + 0.574920i & 635 \\ \hline \end{array}$	0.781500 + 0.476155i	
$\begin{array}{c} 0.9023322701562 \\ 0.076280 + 0.899102i \\ 0.9023322701562 \\ 0.076280 - 0.899102i \\ 0.8645277744797 \\ 0.382051 + 0.775529i \\ 0.8645277744797 \\ 0.382051 - 0.775529i \\ 506 \\ 0.8476860053224 \\ -0.698190 - 0.480732i \\ -0.698190 + 0.480732i \\ 0.8142302777684 \\ -0.299274 + 0.757236i \\ -0.299274 - 0.757236i \\ -0.798620 + 0.118756i \\ 0.8074012496840 \\ -0.798620 - 0.118756i \\ 0.7827062968769 \\ 0.591660 + 0.512414i \\ 0.7827062968769 \\ 0.591660 - 0.512414i \\ -0.094599 - 0.764741i \\ -0.094599 + 0.764741i \\ -0.09$		0.9151315027981
$\begin{array}{c} 0.076280 + 0.899102i & 379 \\ 0.9023322701562 \\ 0.076280 - 0.899102i & 379 \\ 0.382051 + 0.775529i & 506 \\ 0.8645277744797 \\ 0.382051 - 0.775529i & 506 \\ 0.8476860053224 \\ -0.698190 - 0.480732i & 076 \\ 0.8476860053224 \\ -0.698190 + 0.480732i & 076 \\ 0.8142302777684 \\ -0.299274 + 0.757236i & 895 \\ 0.8142302777684 \\ -0.299274 - 0.757236i & 895 \\ 0.8074012496840 \\ -0.798620 + 0.118756i & 946 \\ 0.798620 - 0.118756i & 946 \\ 0.798620 - 0.118756i & 946 \\ 0.798620 - 0.512414i & 492 \\ 0.7705696647074 \\ -0.094599 - 0.764741i & 249 \\ 0.7705696647074 \\ -0.094599 + 0.764741i & 249 \\ 0.7626309295262 \\ -0.501071 + 0.574920i & 635 \\ \end{array}$	0.781500 - 0.476155i	14
$\begin{array}{c} 0.9023322701562 \\ 0.076280 - 0.899102i \\ \hline \\ 0.382051 + 0.775529i \\ \hline \\ 0.382051 - 0.775529i \\ \hline \\ 0.382051 - 0.775529i \\ \hline \\ 0.698190 - 0.480732i \\ \hline \\ -0.698190 + 0.480732i \\ \hline \\ -0.299274 + 0.757236i \\ \hline \\ -0.299274 - 0.757236i \\ \hline \\ -0.798620 + 0.118756i \\ \hline \\ -0.798620 - 0.118756i \\ \hline \\ 0.591660 + 0.512414i \\ \hline \\ -0.591660 - 0.512414i \\ \hline \\ -0.094599 + 0.764741i \\ \hline \\ -0.094599 +$		0.9023322701562
$\begin{array}{c} 0.076280 - 0.899102i & 379 \\ 0.382051 + 0.775529i & 506 \\ 0.8645277744797 \\ 0.382051 - 0.775529i & 506 \\ 0.8476860053224 \\ -0.698190 - 0.480732i & 076 \\ 0.8476860053224 \\ -0.698190 + 0.480732i & 076 \\ 0.8142302777684 \\ -0.299274 + 0.757236i & 895 \\ -0.299274 - 0.757236i & 895 \\ 0.8074012496840 \\ -0.798620 + 0.118756i & 946 \\ 0.798620 - 0.118756i & 946 \\ 0.7827062968769 \\ 0.591660 + 0.512414i & 492 \\ 0.7827062968769 \\ 0.591660 - 0.512414i & 492 \\ 0.7705696647074 \\ -0.094599 - 0.764741i & 249 \\ 0.7705696647074 \\ -0.094599 + 0.764741i & 249 \\ 0.7626309295262 \\ -0.501071 + 0.574920i & 635 \\ \end{array}$	0.076280 + 0.899102i	379
$\begin{array}{c} 0.382051 + 0.775529i \\ 0.382051 + 0.775529i \\ 0.382051 - 0.775529i \\ 0.8645277744797 \\ 506 \\ 0.8645277744797 \\ 506 \\ 0.8476860053224 \\ -0.698190 - 0.480732i \\ -0.698190 + 0.480732i \\ 0.698190 + 0.480732i \\ -0.299274 + 0.757236i \\ 0.8142302777684 \\ -0.299274 - 0.757236i \\ 895 \\ 0.8074012496840 \\ -0.798620 + 0.118756i \\ 0.8074012496840 \\ -0.798620 - 0.118756i \\ 0.7827062968769 \\ 0.591660 + 0.512414i \\ 0.7827062968769 \\ 0.591660 - 0.512414i \\ -0.094599 - 0.764741i \\ 249 \\ 0.7705696647074 \\ -0.094599 + 0.764741i \\ 249 \\ 0.7626309295262 \\ -0.501071 + 0.574920i \\ 635 \\ \end{array}$		0.9023322701562
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$\begin{array}{c} 0.8645277744797 \\ 0.382051 - 0.775529i \\ \hline \\ 0.8476860053224 \\ -0.698190 - 0.480732i \\ -0.698190 + 0.480732i \\ \hline \\ -0.698190 + 0.480732i \\ \hline \\ -0.299274 + 0.757236i \\ -0.299274 - 0.757236i \\ \hline \\ -0.798620 + 0.118756i \\ \hline \\ -0.798620 - 0.118756i \\ \hline \\ 0.8074012496840 \\ -0.798620 - 0.118756i \\ \hline \\ 0.7827062968769 \\ 0.591660 + 0.512414i \\ \hline \\ -0.094599 - 0.764741i \\ \hline \\ -0.094599 + 0.764741i \\ \hline \\ -0.094599 - 0.7626309295262 \\ -0.501071 + 0.574920i \\ \hline \end{array}$		0.8645277744797
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$\begin{array}{c} 0.8476860053224 \\ -0.698190 - 0.480732i \\ -0.698190 + 0.480732i \\ 0.8476860053224 \\ -0.698190 + 0.480732i \\ 0.8142302777684 \\ -0.299274 + 0.757236i \\ 895 \\ 0.8142302777684 \\ -0.299274 - 0.757236i \\ 895 \\ 0.8074012496840 \\ -0.798620 + 0.118756i \\ -0.798620 - 0.118756i \\ 0.8074012496840 \\ -0.798620 - 0.118756i \\ 946 \\ 0.7827062968769 \\ 0.591660 + 0.512414i \\ 492 \\ 0.7827062968769 \\ 0.591660 - 0.512414i \\ 492 \\ 0.7705696647074 \\ -0.094599 - 0.764741i \\ 249 \\ 0.7705696647074 \\ -0.094599 + 0.764741i \\ 249 \\ 0.7626309295262 \\ -0.501071 + 0.574920i \\ 635 \\ \end{array}$		0.8645277744797
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		0.7626309295262
0.7626309295262	-0.501071 + 0.574920i	635
[, 0_000, _000]		0.7626309295262
-0.501071 - 0.574920i 635	-0.501071 - 0.574920i	635



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	0.7418598559614			
0.698543 + 0.249787i	835			
	0.7418598559614			
0.698543 - 0.249787i	835			
	0.7307100992644			
0.730710	868			
	0.7095264289788			
0.157965 - 0.691719i	632			
	0.7095264289788			
0.157965 + 0.691719i	632			
	0.6988568629458			
-0.675796 + 0.178048i	449			
	0.6988568629458			
-0.675796 - 0.178048i	449			
	0.5720459927695			
-0.572046	884			
	0.0642421882346			
0.064242	5668			
No root lies outside the unit circle.				
VAR satisfies the stability condition.				

Figure: 1
Inverse Roots of AR Characteristic Polynomial



Examination of the Stability of the VAR Model

The Table presents the roots and respective modulus of each of the roots in A(L)



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It is observed that

- i. some of the eigen values are positive.
- ii. sone of the eigen values are negative.
- iii. The VAR model is stable (all roots inside the unit circle), it suggests the variables were likely differenced to achieve stationarity before estimating the VAR, as non-stationary variables can lead to spurious results in a VAR unless cointegrated.
- iv. Again, all inverse roots must lie inside the unit circle (i.e., their modulus or absolute value must be less than 1) which means that the VAR model satisfies the stability condition.
- v. The red dot, being close to the unit circle (but still inside), might suggest a root with a modulus near 1, indicating a relatively persistent dynamic, though still stable.

Summary Conclusion and Policy Implications: Granger causality analysis suggested a unidirectional causal relationship from exchange rate to imports and a bidirectional causal relationship between exports and imports in the Kazakhstan economy. This means that changes in the exchange rate influence import levels, while exports and imports mutually affect each other, reflecting their interdependence.

Again, the Johansen Cointegration Analysis confirmed the existence of cointegration among exchange rate, exports, and imports at levels over the period from November 1993 to October 2022. This indicates a stable long-run relationship among these variables, implying that the exchange rate of currencies relevant to Kazakhstan's trade is systematically linked with its export and import dynamics.

Similarly, the Vector Autoregression (VAR) model estimates further confirmed that the stable long-run relationships among exchange rate, exports, and imports across various lags. Specifically, the exchange rate equation (22) shows that the coefficients for lags 3 and 4 of the exchange rate are significant at the 1% level, indicating that a 1-unit depreciation in the exchange rate three or four months prior significantly affects the current exchange rate, reflecting persistent depreciation effects in the Kazakhstan economy. Additionally, the coefficient for lag 9 of imports is highly significant, suggesting that a 1-unit increase in imports nine months prior contributes to exchange rate depreciation. Similarly, the coefficients for lags 6 and 9 of exports are significant at the 1% level, indicating that higher exports reduce exchange rate depreciation, leading to currency appreciation. This reflects increased foreign currency inflows from export growth, which stabilizes the exchange rate.

In import equation (23), the coefficient for lag 1 of imports is significant at the 1% level, indicated that a 1-unit increase in imports one month prior reduces current import growth, suggesting mean-reversion in import dynamics, possibly due to budget constraints or trade adjustments. The coefficients for lags 1 and 8 of exports are also significant at the 1% level, highlighting a linkage between export revenues and import demand. Sustained export growth supports higher imports, reflecting Kazakhstan's trade dynamics.

Finally, the export equation (24) confirmed that the coefficients for lags 1, 2, 7, 8, and 9 of exports are significant, reinforcing mean-reversion in export growth. Additionally, the coefficients for lags 4, 5, 8, and 9 of imports are significant, showing that a 1-unit increase in imports up to nine months prior increases exports. This relationship likely reflects the role of imported inputs in boosting Kazakhstan's oil production, a key driver of its export sector.



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Policy Implications:

Policymakers can use the model to simulate the impact of exchange rate interventions (e.g., devaluation) on trade balances. For instance, a weaker tenge could boost exports but harm import-dependent sectors. The model can forecast future trade flows or exchange rate movements based on historical patterns. The negative covariance between exchange rate and imports suggested that a managed depreciation could reduce import dependence, supporting domestic industries. The positive import-export covariance indicated that policies boosting exports (e.g., oil production) may increase imports, affecting the trade balance.

The negative covariances with imports and exports highlight the exchange rate's role in trade. A depreciating tenge (higher exchange rate) reduces imports due to higher costs, but the negative covariance with exports may reflect the dominance of USD-priced oil exports, which are less sensitive to exchange rate changes in volume terms. The large positive covariance between imports and exports underscores the interdependence of Kazakhstan's trade flows. For example, importing capital goods (e.g., drilling equipment) supports oil production, which drives exports. The large variances for imports and exports reflect Kazakhstan's exposure to global commodity price shocks and trade disruptions, common in resource-dependent economies.

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