

Value of π , a Mathematical Approach

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Abstract

This paper discusses about the exact value of π is $(17 - 8\sqrt{3})$ and approximate value of π that is 3.14359353944... This paper shows that the values of pi derived geometrically and algebraically are the same.

Key words: simple pi, value of pi, tricky pi.

Introduction:

There are some mathematical questions in the world that remain unsolved for years together. Amongst them one of the questions is the exact value of the π . So I started trying to solve that problem. However, the same answer was found by the geometric and algebraic methods. Thus, this value of pi could be looked upon as the new value of Pi.

As we know, the value of π was discovered around 4000 years ago. Many experts tried to find the right answer by studying different methods. But so far no one was able to give the perfect solution, which is not strange. Current value of pi is 3.141592653... with the help of a calculator around 62.8 trillion digits. So many digits have been found out but still exploring the next values. The value of pi is approximate; it is said that it cannot be exact.

In mathematics there are two types of values: Exact and Approximate. For example,

Exact value	Approximate value
1/3	= 0.33333...
2/3	= 0.66666...
$\sqrt{2}$	= 1.41421...
$\sqrt{3}$	1.73205...

Two types of prices are given above namely Exact and Approximate. From the above table, a line can be drawn as long as this exact value and a line of approximate length cannot be drawn.

For example, we will be able to show a line of 1/3 rather than the line of its approximate value 0.33333... Even the line of length $\sqrt{2}$ and $\sqrt{3}$ can be displayed rather than 1.41421..., 1.73205... Their approximate values respectively.

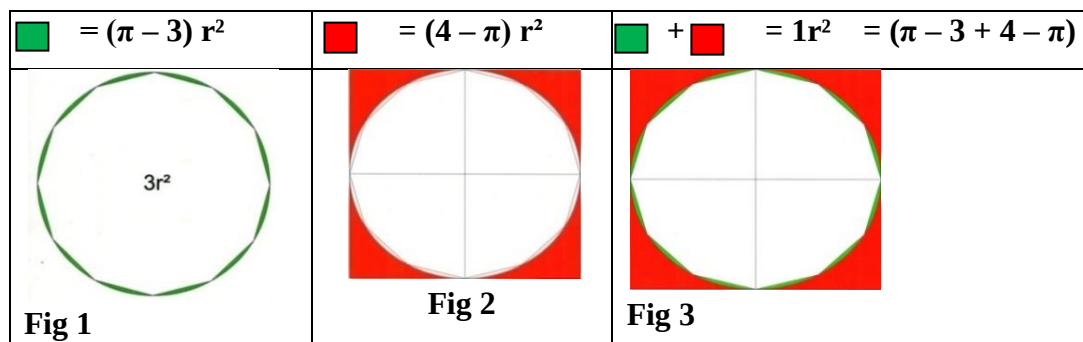
To draw a line of length equal to the value of pi π i.e. the distance in one revolution of the wheel of the cart (diameter $\times \pi$) is a line of length of pi π . In many rounds (rounds $\times$ diameter $\times \pi$) such a distance (rounds $\times$ diameter) is a line of length of pi π . From this pi π must have an exact value.

The reason why it was said that the value of pi π cannot be exact is because of the method (n sides polygon method) used to find it. Although this method produces polygons with infinite sides, it is assumed that there is no end in the middle, and thus the pi π cannot have an exact value.

If the value of pi π is not exact, I have got more equations that don't have exact answers to the currently accepted value of pi π .

One of the examples is; If $(\pi - 3) = 0.141592653\dots$ and $(4 - \pi) = 0.858407346\dots$ then $(4 - \pi) + (\pi - 3) = 1 = 0.999999999\dots$ but it shows that $(4 - \pi) + (\pi - 3) \neq 1$. But this is wrong.

You can observe from the given figure;



There are many equations based on such geometric figures. Algebraic equations are so infinite that they cannot be given exact answers with the help of the present universal value of pi π . Also because of the current value of pi, it is said that it is impossible to get the square equivalent to the area of the circle by itself. But check out this easy-to-build example that shows it is possible.

$$\text{Yellow square} = \pi r^2 \quad \text{Red square} = 2r^2 \quad \text{Yellow square} = \pi r^2 \quad \text{Red square} = 2r^2$$

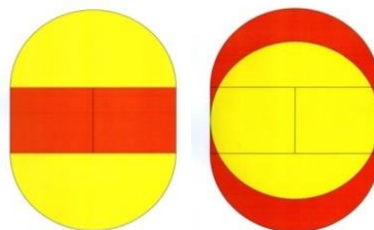


Fig 4

fig 5

This color code area mapping is possible. We can find circle whose area equal to the area of square as well as square whose area equal to the area of circle. This transformation is possible for algebraic value of π .

Consider an extremely simple and elegant geometric relationship for deriving the area of a circle:

$$\text{Area of circle} = (\text{Area of 1 inscribed dodecagon} + \text{area of 2 circumscribed dodecagons}) = 3(17 - 8\sqrt{3}) r^2$$

This equation suggests an exact value for π , which has been validated through multiple derivations. However, this formula also reveals a previously unnoticed discrepancy that arises when using the approximate value of π , confirming the presence of an inherent error in traditional measurement-based methods.

This expression extends infinitely, leading to new insights in geometric calculations. If the radius of a circle is $1r$, then its diameter is $2r$. Based on this, a square of dimensions $2r \times 2r$ is initially constructed. Within this square, the area occupied by the circle is πr^2 . The portions of the square outside the circle are then analysed, and some of these segments are illustrated in the following figures.

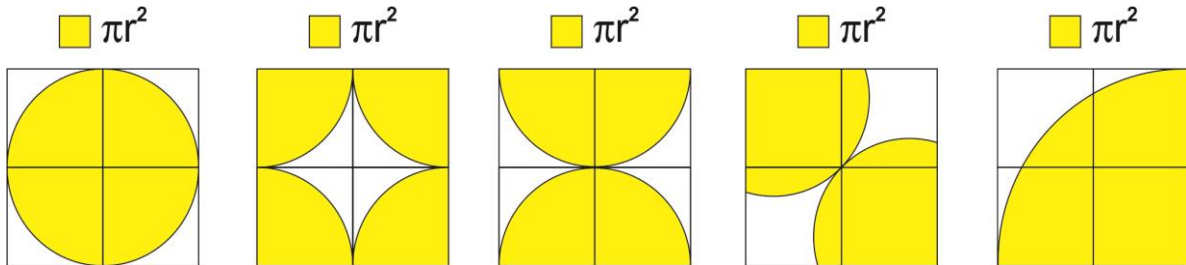


Fig 6

Fig 7

Fig 8

Fig 9

Fig 10

Analysis of Area Distribution in Circle Segmentation

If specific points within the given circles are connected, certain regions within the circles align, forming identifiable areas. However, some portions remain unaccounted for. These variations in area distribution are illustrated in the following figures.

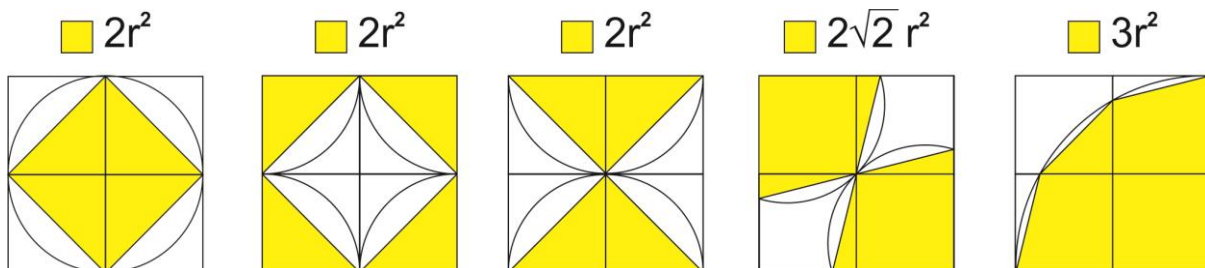


Fig 11

Fig 12

Fig 13

Fig 14

Fig 15

In Figures 11, 12, and 13, the circular region with an area of $2r^2$ was identified, while a portion of the circular area remained unaccounted for. Subsequently, in Figure 14, an area of $2\sqrt{2}r^2$ was determined. Furthermore, in Figure 15, an area of $3r^2$ was obtained. Since only a small portion of the circular area remained unaccounted for, an analysis of the remaining area in Figure 15 was initiated to determine its exact measure.

As shown in the below fig.

: Geometric approach

Let us consider the $(1/4)$ th part of the circle whose radius is $r = 2$. Then area of circle whose radius r is $2 = 4\pi r^2$. Thus, area of $(1/4)$ th part of that circle $= 4\pi r^2/4 = \pi r^2$. Moreover the area of 12 side polygon formed in this circle $= 4 \times 3r^2 = 12r^2$, Therefore, area of $(1/4)$ th part of dodecagon $= 12r^2/4 = 3r^2$ as shown in the below figure 11 and 12 respectively

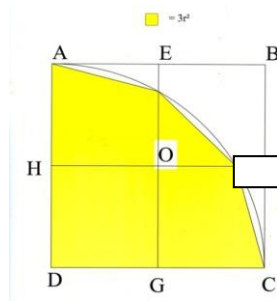


Fig 16,

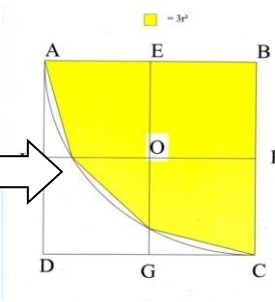


fig. 17

Hence the area of yellow colour part is $3r^2$ and area of white colour part = r^2 . After joining the appropriate points of the above figure 16 and 17. We get, the new geometry shown in figure 18 and 19 respectively.

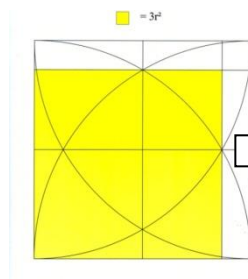


Fig 18,

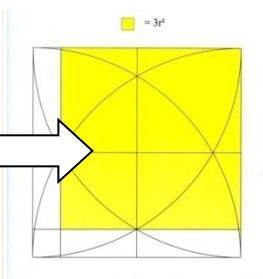


fig. 19

The area of yellow colour square in figure 18 and figure 19 = $3r^2$ and the length of the side of the square = $\sqrt{3}r$. Thus, area of square ($\sqrt{3}r \times \sqrt{3}r$) = $3r^2$. Now we are overlapping figure 18 and figure 19 on each other. Then, we get figure 20 and figure 21.

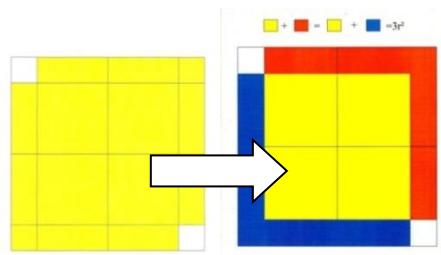


Fig 20,

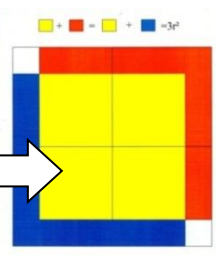


fig. 21

From figure 21, Area of yellow colour part + area of red colour part = area of yellow colour part + area of blue colour part = $3r^2$. Further, if we add area of white colour squares opposite to each other in dimension $3r^2$. Then we will get area of circle. Hence the area of white colour squares is nothing but the remaining parts of circle.

So the side of the white square = $(2 - \sqrt{3})r$

$\therefore$ Area of the white colour square = $[(2 - \sqrt{3})r]^2 = (7 - 4\sqrt{3})r^2$

$\therefore$ Area of two white colour square = $2(7 - 4\sqrt{3})r^2 = (14 - 8\sqrt{3})r^2$

If we add $3r^2$ in above formula we get, Area of circle = $3r^2 + (14 - 8\sqrt{3})r^2$

Therefore, Area of circle = $(17 - 8\sqrt{3}) r^2 \Rightarrow \pi = 17 - 8\sqrt{3}$

The supported method is illustrated in Figures 17 and 19.

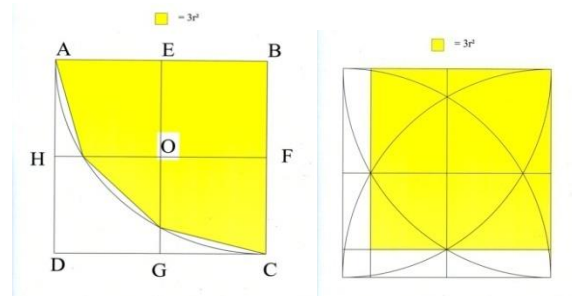


Fig 22,

fig. 23

In Figures 22 and 23, the yellow region represents an area of $3r^2$, while the white region corresponds to $(4r^2 - 3r^2) = 1r^2$. In this approach, subtracting the portion outside the circle from the square region results in an area of $1r^2$. Within this $1r^2$, a small portion of the circular region remains unaccounted for, requiring further investigation to determine its exact measure. In Figures 22 and 23, the white region is highlighted in red, leading to the formation of Figures 24 and 25. In Figures 24 and 25, distinct regions of $1r^2$ and $3r^2$ can be observed. If the $1r^2$ region in Figure 25 is duplicated using the L-type pattern, the remaining regions still include $1r^2$ and $3r^2$. Therefore, the remaining portion of the circle in this L-type pattern must be analysed to determine its exact measure.

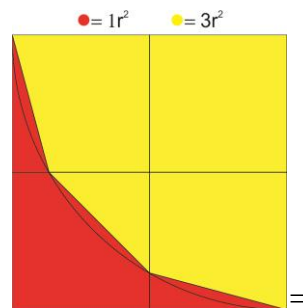


Fig 24,

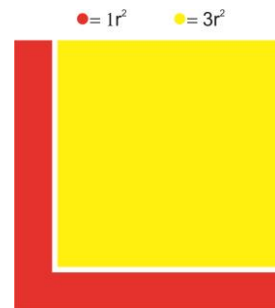


fig. 25

A square of area $3r^2$ was placed on the L-type pattern, as shown in Figure 26. The next step is to analyse the proportion of the remaining region within the L-type pattern.

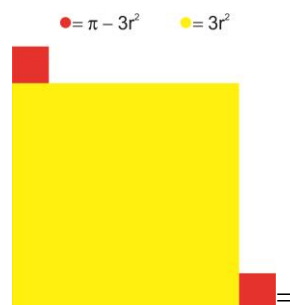


Fig 26,

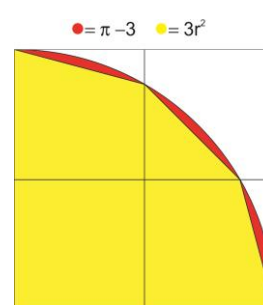


fig. 27

In Figure 26, the yellow region represents an area of $3r^2$. The side length of the square is $\sqrt{3}r$, while the width of the L-type pattern is $(2 - \sqrt{3})r$. The two adjacent squares within the L-type pattern have a combined area given by:

$$(2r - \sqrt{3}r)^2 \times 2 = 2(7 - 4\sqrt{3})r^2 = (14 - 8\sqrt{3})r^2$$

This represents the remaining portion of the circular region. Further analysis is required to determine this portion accurately.

Following this approach, the square is placed on the L-type pattern. Observing the back side of Figure 21 will reveal its corresponding form in Figure 28.

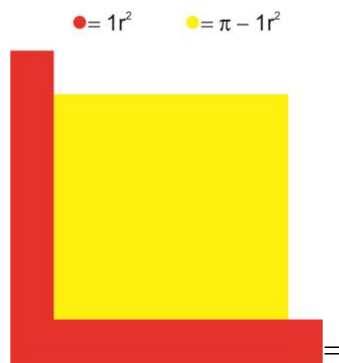


Fig 28

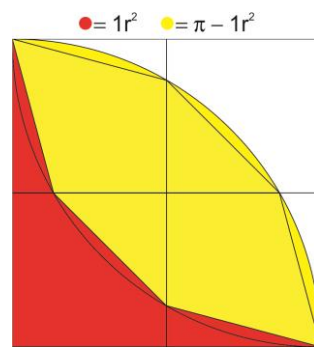


Fig 29

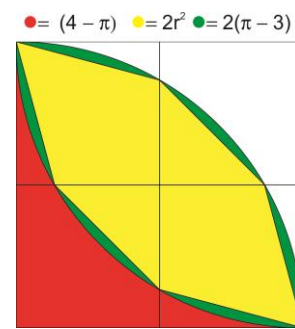


Fig 30

In Figure 28, the L-type pattern has an area of $1r^2$, while in Figure 29; the red region represents a different area. The yellow region in Figure 28 has an area given by:

$$[\sqrt{3}r - (2 - \sqrt{3})r] = (2\sqrt{3} - 2)^2 r^2 = (16 - 8\sqrt{3})r^2$$

In Figure 29, the total area of the yellow region is:

$$2r^2 + \text{remaining circular portion} = (2 + 14 - 8\sqrt{3})r^2$$

Thus, in Figures 28 and 29, the red-marked area corresponds to the unaccounted portion of the circle, which equals the yellow region's area. Therefore, using both methods, the remaining circular area is:

$$(14 - 8\sqrt{3})r^2$$

Adding the previously obtained $3r^2$, the total circular area becomes:

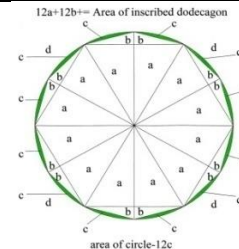
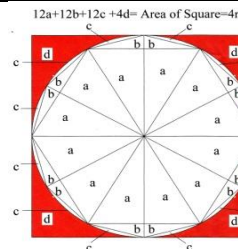
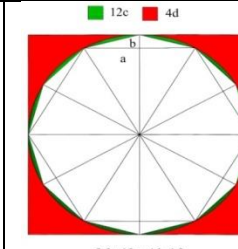
$$3r^2 + (14 - 8\sqrt{3})r^2$$

$$= (17 - 8\sqrt{3})r^2$$

Supported work

Algebra proofs

Basic information: **Note:** let a, b, c & d each part shows area in following figures

 12a+12b = Area of inscribed dodecagon area of circle = 12c	 12a+12b+12c+4d = Area of Square = 4r² Area of Circle = 4d	 12c 4d 3r² + 12c + 4d = 4r²
Area of inscribed dodecagon = (12a + 12b) = 3r² = (πr² - 12c)	Area of circumscribed square = (12a + 12b + 12c + 4d) = 4r² = (πr² + 4d)	(4 - π) r² = 4d (π - 3) r² = 12c (4d + 12c) = [(4 - π) + (π - 3)] = 1

Area of inscribed hexagon = 12a Area of inscribed dodecagon = (12a + 12b) (12a + 12b) - 12a = 12b 0.125√3) r² = (πr² + 4d) = (3r² + 12c) = (4r² - 4d) 0.25r² = a + b	Area of circumscribed square = 4r² Area of circle = 12c (a + b - 3c - d) = 0	= (1.5√3) r² a = (0.125√3) r² = 3r² = (πr² - 12c) = (3 - 1.5√3) r² b = (0.25 - = (12a + 12b + 12c + 4d) = 4r² = (12a + 12b + 12c) (12c + 4d = 1r²)/4 = (3c + d =
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As we know, the exact area of inscribed dodecagon = 3r². In order to calculate exact area of circle, we have to calculate exact area of 12c. Hence there is no need to divide whole circle into infinite number of parts to calculate its accurate area. How to estimate the exact area of part 12c & part 4d?

How I solve problem 12c & 4d let us see

$$12c = (\pi - 3)r^2 \text{ \& } 4d = (4 - \pi) r^2$$

I found some Area of circumscribed square equations. I have geometric proofs for all of the following equations.

$$x(\text{Area of circumscribed square}) = x(4r^2)$$

S. r.	Equations of area of square	= ... area of circumscribed squares
1	1(12a + 12b + 12c + 4d)	= (12a + 12b + 12c + 4d)
2	2(16a + 16b)	= (32a + 32b)
3	3(64b + 12c + 8d)	= (192b + 36c + 24d)
4	4(18a + 2b + 3c)	= (72a + 8b + 12c)
5	5(96b - 6c + 4d)	= (480b - 30c + 20d)
6	6(4a + 68b + 4d)	= (24a + 408b + 24d)
7	7(-8a + 120b + 8d)	= (-56a + 840b + 56d)
8	8(20a - 12b + 6c)	= (160a - 96b + 48c)
9	9(128b - 24c)	= (1152b - 216c)

10	$10(16d + 48c)$	$= (160d + 480c)$
	Total area of 55 square = $55(4r^2)$	$= (244a + 3028b + 342c + 288d)$

Use above equations and found area of circle.

Area of circumscribed 55 square = $(244a + 3028b + 342c + 288d)$ use equation $(a + b - 3c - d = 0)$

$$= (244a + 3028b + 342c + 288d) + 114(a + b - 3c - d)$$

$$= (244a + 3028b + 342c + 288d) + (114a + 144b - 342c - 114d)$$

$$= (358a + 3142b + 174d)$$

$$= 358a + 3142b + [174d = 43.5(4r^2 - \pi r^2)]$$

$$= (358a + 3142b) + (174r^2 - 43.5\pi r^2)$$

$$\text{Area of circumscribed 55 square} + 43.5\pi r^2 = (358a + 3142b + 174r^2)$$

$$\text{Area of } 43.5\pi r^2 = (358a + 3142b + 174r^2) - 55(4r^2)$$

$$= (358a + 3142b + 174r^2) - 220r^2$$

$$= 358(0.125\sqrt{3}) r^2 + 3142(0.25 - 0.125\sqrt{3}) r^2 - 46r^2$$

$$= (44.75\sqrt{3}) r^2 + (785.5 - 392.75\sqrt{3}) r^2 - 46r^2$$

$$= (739.5 - 348\sqrt{3}) r^2$$

$$\text{Area of circle} = (739.5 - 348\sqrt{3}) r^2 / 43.5$$

$$= (17 - 8\sqrt{3}) r^2$$

Area of circumscribed 55 square – Area of circumscribed 28.5 square

$$= (244a + 3028b + 342c + 288d) - 28.5(12a + 12b + 12c + 4d)$$

$$= \text{area of circumscribed 26.5 square} = (244a + 3028b + 342c + 288d) - (342a + 342b + 342c + 114d)$$

$$= (-98a + 2686b + 174d)$$

$$= [-98a + 2686b + 43.5(4 - \pi r^2)]$$

$$= [-98a + 2686b + 174r^2 - 43.5\pi r^2]$$

$$\text{Area of 26.5 square} + 43.5\pi r^2 = (-98a + 2686b + 174r^2)$$

$$\text{Area of } 43.5\pi r^2 = (-98a + 2686b + 174r^2) - 26.5(4r^2)$$

$$= \text{Area of } 43.5\pi r^2 = (-98a + 2686b + 174r^2) - (106r^2)$$

$$= -98(0.125\sqrt{3})r^2 + 2686(0.25 - 0.125\sqrt{3})r^2 + 68r^2$$

$$= (-12.25\sqrt{3})r^2 + (671.5 - 335.75\sqrt{3})r^2 + 68r^2$$

$$= (739.5 - 348\sqrt{3})r^2$$

$$\text{Area of circle} = (739.5 - 348\sqrt{3})r^2 / 43.5 = (17 - 8\sqrt{3})r^2$$

$$= [244a + 3028b + 28.5(\pi - 3)r^2 + 72(4 - \pi)r^2]$$

$$= (244a + 3028b + (28.5\pi - 85.5)r^2 + (288 - 72\pi)r^2]$$

$$= (244a + 3028b + (202.5 - 43.5\pi)r^2$$

$$= 244(0.125\sqrt{3})r^2 + 3028(0.25 - 0.125\sqrt{3})r^2 + (202.5r^2 - 43.5\pi r^2)$$

$$= (30.5\sqrt{3})r^2 + (757 - 378.5\sqrt{3})r^2 + (202.5r^2 - 43.5\pi r^2)$$

$$= (959.5 - 348\sqrt{3})r^2 - 43.5\pi r^2$$

$$\text{Area of 55 square} + 43.5\pi r^2 = (959.5 - 348\sqrt{3})r^2$$

$$\begin{aligned} \text{Area of } 43.5\pi r^2 &= (959.5 - 348\sqrt{3})r^2 - 55(4r^2) &= (959.5 - 348\sqrt{3})r^2 - (220r^2) \end{aligned}$$

$$\begin{aligned} \text{Area of circle} &= (739.5 - 348\sqrt{3})r^2 / 43.5 \\ &= (17 - 8\sqrt{3})r^2 \end{aligned}$$

Ten equations were given. Among them, some equations and a few newly derived equations were obtained. By multiplying these equations with different numbers, perfect squares were formed.

From the table of these squares, observe how the sum of all the squares leads to the question involving **C and D**, and how that question was solved using different methods. Also, see how the **area of the circle** was determined from it.

Thousands of such examples were tested, yet the **area of the circle** always turned out to be the same. Hence, I confirm that the **area of the circle is $(17 - 8\sqrt{3})r^2$** .

S. r. no.	Equations of area of square = $4r^2$	No. of square	Part a	Part b	Part c	Part d
1	$12a+12b+12c+4d$	*1	12a	12b	12c	4d
2	$16a+16b$	*2	32a	32b		
3	$64b+12c+8d$	*3		192b	36c	24d
4	$18a+2b+3c$	*4	72a	8b	12c	
5	$28a - 68b+18c$	*5	140a	-340b	90c	
6	$96b - 6c+4d$	*6		576b	-36c	24d

7	128b – 24c	*7		896b	-168c	
8	20a – 12b+6c	*8	160a	-96b	48c	
9	8a+72b – 12c	*9	72a	648b	-108c	
10	-8a+120b+8d	*10	-80a	1200b		80d
11	12a+44b – 6c	*11	132a	484b	-66c	
12	4a+100b – 18c	*12	48a	1200b	-216c	
13	24a – 40b+12c	*13	312a	-520b	156c	
14	13a+29b+d	*14	182a	406b		14d
15	48c+16d	*15			720c	240d
Total square & part		*120	1082a	4698b	480c	386d

Area of circumscribed 120 square = $(1082a + 4698b + 480c + 386d)$ use equation $(a + b - 3c - d = 0)$

$$= (1082a + 4698b + 480c + 386d) + 160(a + b - 3c - d)$$

$$= (1082a + 4698b + 480c + 386d) + (160a + 160b - 480c - 160d)$$

$$= (1242a + 4858b + 226d)$$

$$= 1242a + 4858b + [226d = 56.5(4r^2 - \pi r^2)]$$

$$= (1242a + 4858b) + (226r^2 - 56.5\pi r^2)$$

$$\text{Area of circumscribed 120 square} + 56.5\pi r^2 = (1242a + 4858b + 226r^2)$$

$$\text{Area of } 56.5\pi r^2 = (1242a + 4858b + 206r^2) - 120(4r^2)$$

$$= (1242a + 4858b + 226r^2) - 480r^2$$

$$= 1242(0.125\sqrt{3}) r^2 + 4858(0.25 - 0.125\sqrt{3}) r^2 - 254r^2$$

$$= (155.25\sqrt{3}) r^2 + (1214.5 - 607.25\sqrt{3}) r^2 - 254r^2$$

$$= (960.5 - 452\sqrt{3}) r^2$$

$$\pi r^2 = (960.5 - 452\sqrt{3}) r^2 / 56.5 = (17 - 8\sqrt{3}) r^2$$

S. r. no.	Equations of area of square = $4r^2$	No. of square	Part a	Part b	Part c	Part d
1	12a+12b+12c+4d	*30	360a	360b	360c	120d
2	16a+16b	*29	464a	464b		
3	64b+12c+8d	*28		1792b	336c	224d
4	18a+2b+3c	*27	486a	54b	81c	
5	28a – 68b+18c	*26	728a	-1768b	468c	

6	$96b - 6c + 4d$	*25		2400b	-150c	100d
7	$128b - 24c$	*24		3072b	-576c	
8	$20a - 12b + 6c$	*23	460a	-276b	138c	
9	$8a + 72b - 12c$	*22	176a	1584b	-264c	
10	$-8a + 120b + 8d$	*21	-168a	2520b		168d
11	$12a + 44b - 6c$	*20	240a	880b	-120c	
12	$4a + 100b - 18c$	*19	76a	1900b	-342c	
13	$24a - 40b + 12c$	*18	432a	-720b	216c	
14	$13a + 29b + d$	*17	221a	493b		17d
15	$48c + 16d$	*16			768c	256d
Total square & part		*345	3475a	12755b	915c	885d

Area of circumscribed 345 square = $(3475a + 12755b + 915c + 885d)$ use equation $(a + b - 3c - d = 0)$

$$= (3475a + 12755b + 915c + 885d) + 305(a + b - 3c - d)$$

$$= (3475a + 12755b + 915c + 885d) + (305a + 305b - 915c - 305d)$$

$$= (3780a + 13060b + 580d)$$

$$= 3780a + 13060b + [580d = 145(4r^2 - \pi r^2)]$$

$$= (3780a + 13060b) + (580r^2 - 145\pi r^2)$$

$$\text{Area of circumscribed 345 square} + 145\pi r^2 = (3780a + 13060b + 580r^2)$$

$$\text{Area of } 145\pi r^2 = (3780a + 13060b + 580r^2) - 345(4r^2)$$

$$= (3780a + 13060b + 580r^2) - 1380r^2$$

$$= 3780(0.125\sqrt{3}) r^2 + 13060(0.25 - 0.125\sqrt{3}) r^2 - 800r^2$$

$$= (472.5\sqrt{3}) r^2 + (3265 - 1632.5\sqrt{3}) r^2 - 800r^2$$

$$= (2465 - 1160\sqrt{3}) r^2$$

$$\pi r^2 = (2465 - 1160\sqrt{3}) r^2 / 145 = (17 - 8\sqrt{3}) r^2$$

Conclusion

Area of circle = $(17 - 8\sqrt{3}) r^2$ is proved by Algebraic equations and Geometric constructions. Thus, it can be concluded that the value of pi stands out to be $17 - 8\sqrt{3}$.

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