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Computation of Atom Bond Sum Connectivity Downhill Indies Of Graphs

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Abstract - In This Study, We Calculate The Downhill Abs Indices For A Graph. Additionally, We Compute These Indices For Some Usual Types Of Graphs, (Such As R-Regular Graphs, Path Graphs, Complete Bipartite Graphs) Wheel Graphs, Helm Graphs, And Gear Graphs.

Key Words - Downhill Abs Index, Wheel Graph, Gear Graph, Helm Graph.

1. Introduction –

In This Study, G Refers To Both V(G) And E(G), Which Are The Sets Of Vertices And Edges Of The Graph G. The Degree Of A Vertex U, Denoted As $d_G(u)$, Is The Number Of Vertices Connected To U. Any Terms Or Symbols That Are Not Defined Here Can Be Found In [1].

A Topological Index Is A Numerical Value That Is Calculated Based On The Structure Of The Graph.

A Path From Vertex U To Vertex V In Graph G Is A Sequence Of Vertices That Begins With U And Ends With V, S.T. Consecutive Vertices In P Are Adjacent, And Each Vertex Appears Only Once. A Path $\pi = v_1, v_2, \cdots v_{k+1}$ In G Is A Downhill Path If For Every J, 1≤ J ≤ K, $d_G(v_j) \geq d_G(v_{j+1})$.

A Vertex V Is Said To Downhill Dominate A Vertex U If There Is A Downhill Path That Starts At U And Leads To V. The Downhill Neighborhood Of A Vertex V Is Denoted By Absd(V) And Is Defined As:

$ABSD(V) = \{\text{U: V Downhill Dominates U}\}$

The Downhill Degree $d_{dn}(v)$ Of A Vertex V Is The No, Of Downhill Neighbors Of V [2].

The Atom Bond Sum Connectivity (Abs) Index Was Presented In [3]And It Is Defined As

$$ABS(G) = \sum_{uv \in E(G)} \frac{d_G(u) + d_G(v) - 2}{d_G(u) + d_G(v)}$$

Recently, Some Abs Indices Were Studied In [4,5].

Motivated By Abs Index, The Abs Downhill Index Of A Graph G Is Defined As

$$ABSD(G) = \sum_{uv\in E(G)} \sqrt{\frac{d_{dn}(u) + d_{dn}(v) - 2}{d_{dn}(u) + d_{dn}(v)}}$$

2. Result For Some Usual Graphs-

Proposition (1) - Let G Be R- Regular With N Vertices And R ≥ 2. Then

$$ABSD(G) = \frac{nr}{2}\sqrt{\frac{(n-2)}{(n-1)}}$$

Proof – Let G Be R- Regular Graph With N Vertices And R ≥ 2 And $\frac{nr}{2}$ Edges.

Then $d_{dn}(v) = n - 1$ For Each V In G.

From Definition

$$ABSD(G) = \sum_{uv\in E(G)} \sqrt{\frac{d_{dn}(u) + d_{dn}(v) - 2}{d_{dn}(u) + d_{dn}(v)}}$$

$$= \frac{nr}{2}\sqrt{\frac{(n-1) + (n-1) - 2}{(n-1) + (n-1)}}$$

$$= \frac{nr}{2}\sqrt{\frac{(n-2)}{(n-1)}}$$

Corollary 1.1. Let Cn Be A Cycle With N ≥ 3 Vertices. Then

$$ABSD(C_n) = n\sqrt{\frac{(n-2)}{(n-1)}}$$

Corollary 1.2. Let K_n Be A Complete Graph With N ≥ 3 Vertices. Then

$$ABSD(K_n) = \frac{n}{2}\sqrt{(n-1)(n-2)}$$

Proposition (2) – Let P_n Be A Path With N ≥ 3 Vertices. Then

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$$ABSD(P_n) = 2\sqrt{\frac{(n-3)}{(n-1)}} + (n-3)\sqrt{\frac{(n-2)}{(n-1)}}$$

Proof – Let P_n Be A Path With N ≥ 3 Vertices. Clearly, P_n Has Two Types Of Edges Based On The Downhill Degree Of End Vertices Of Edges As Follows:

$E_1 = \{Uv \in E(P_n) | D_{Dn}(U) = 0,\ D_{Dn}(V) = N-1\}, |E_1| = 2$

$E_2 = \{Uv \in E(P_n) | D_{Dn}(U) = D_{Dn}(V) = N-1\}, |E_2| = N-3$

$$ABSD(P_n) = \sum_{uv \in E(P_n)} \sqrt{\frac{d_{dn}(u) + d_{dn}(v) - 2}{d_{dn}(u) + d_{dn}(v)}}$$

$$= 2\sqrt{\frac{0 + (n-1) - 2}{0 + (n-1)}} + (n-3)\sqrt{\frac{(n-1) + (n-1) - 2}{(n-1) + (n-1)}}$$

$$= 2\sqrt{\frac{(n-3)}{(n-1)}} + (n-3)\sqrt{\frac{(n-2)}{(n-1)}}$$

Proposition (3) – Let $K_{m,n}$ Be A Complete Bipartite Graph With M<N. Then

$$ABSD(K_{m,n}) = mn\sqrt{\frac{(n-2)}{n}}$$

Proof - We Obtain The Partition Of The Edge Set Of $K_{m,n}$ As Follows:

$$E_1 = \left\{Uv \in E(K_{m,n}) \middle| D_{Dn}(U) = N,\ D_{Dn}(V) = 0\right\}, |E_1| = Mn$$

$$ABSD(K_{m,n}) = mn\sqrt{\frac{n + 0 - 2}{n + 0}}$$

$$ABSD(K_{m,n}) = mn\sqrt{\frac{(n-2)}{n}}$$

3. Results For Wheel Graphs -

The Wheel W_n Is The Connection Of C_n And K_1. Obviously W_n Has 2n Edges And N+1 Vertices. The Vertex K_1 Is Called Apex And The Vertices Of C_n Are Named Rim Vertices.

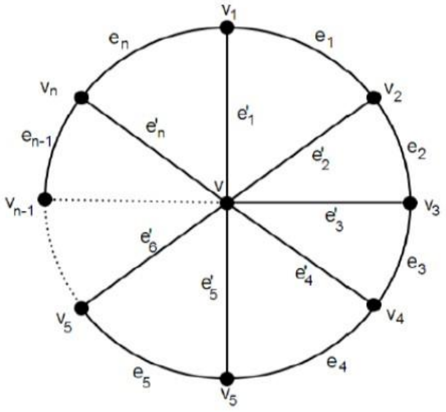

Figure 1. Wheel Graph W_n

Let W_n Be A Wheel With 2n Edges And N+1 Vertices, N ≥ 4. There Are Two Kinds Of Edges, Which Are Determined By The Downhill Degree Of The End Vertices Of Each Edge, As Follows [6].

$$\mathrm{E}_1 = \{\mathrm{Uv} \in \mathrm{E}(W_n) |\, \mathrm{D_{Dn}}(\mathrm{U}) = \mathrm{N},\ \mathrm{D_{Dn}}(\mathrm{V}) = \mathrm{N} - 1\}, |\mathrm{E}_1| = \mathrm{N}$$

$$\mathrm{E}_2 = \{\mathrm{Uv} \in \mathrm{E}(W_n) |\, \mathrm{D_{Dn}}(\mathrm{U}) = \ \mathrm{D_{Dn}}(\mathrm{V}) = \mathrm{N} - 1\}, |\mathrm{E}_2| = \mathrm{N}$$

Formula 1. Let W_n Be A Wheel With (N+1) Vertices And 2n Edges, N ≥ 4. Then

$$ABSD(W_n) = n\sqrt{\frac{(2n-3)}{(2n-1)}} + n\sqrt{\frac{(n-2)}{(n-1)}}$$

Proof – We Deduce

$$ABSD(W_n) = \sum_{uv \in E(P_n)} \sqrt{\frac{d_{dn}(u) + d_{dn}(v) - 2}{d_{dn}(u) + d_{dn}(v)}}$$

$$ABSD(W_n) = n\sqrt{\frac{n + (n-1) - 2}{n + (n-1)}} + n\sqrt{\frac{(n-1) + (n-1) - 2}{(n-1) + (n-1)}}$$

$$= n\sqrt{\frac{(2n-3)}{(2n-1)}} + n\sqrt{\frac{(n-2)}{(n-1)}}$$

4. Result For Helm Graphs -

The Helm Graph H_n Is A Graph Attained From W_n (With (N+1) Vertices) By Attaching An End Edge To Each Rim Vertex Of W_n.

Clearly, $|\mathrm{V}\,(H_n) = 2\mathrm{n} + 1\,|$ And $|E(H_n) = 3n|$

A Graph H_n Is Shown In Figure 2.

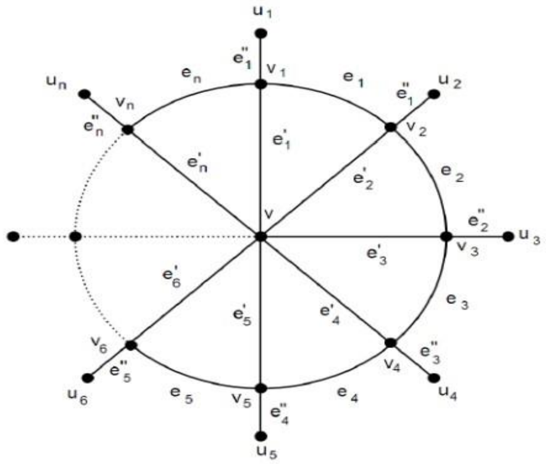

Figure 2. Helm Graph H_n

Let H_n With 3n Edges, N ≥ 3. Then H_n Has Three Kinds Of The Downhill Degree Of Edges As Follows [7]:

$$\mathrm{E}_1 = \{\mathrm{Uv} \in \mathrm{E}(H_n) | \mathrm{D}_{\mathrm{Dn}}(\mathrm{U}) = 2\mathrm{n},\ \mathrm{D}_{\mathrm{Dn}}(\mathrm{V}) = 2\mathrm{n} - 1\}, |\mathrm{E}_1| = \mathrm{N}$$

$$\mathrm{E}_2 = \{\mathrm{Uv} \in \mathrm{E}(H_n) | \mathrm{D}_{\mathrm{Dn}}(\mathrm{U}) = \mathrm{D}_{\mathrm{Dn}}(\mathrm{V}) = 2\mathrm{n} - 1\}, |\mathrm{E}_2| = \mathrm{N}$$

$$\mathrm{E}_3 = \{\mathrm{Uv} \in \mathrm{E}(H_n) | \mathrm{D}_{\mathrm{Dn}}(\mathrm{U}) = 2\mathrm{n} - 1,\ \mathrm{D}_{\mathrm{Dn}}(\mathrm{V}) = 0\}, |\mathrm{E}_3| = \mathrm{N}$$

Formula 2. Let H_n With (2n+1) Vertices, N ≥ 3. Then

$$ABSD(H_n) = n\sqrt{\frac{(4n-3)}{(4n-1)}} + n\sqrt{\frac{2(n-1)}{(2n-1)}} + n\sqrt{\frac{(2n-3)}{(2n-1)}}$$

Proof – We Obtain

$$ABSD(H_n) = \sum_{uv \in E(P_n)} \sqrt{\frac{d_{dn}(u) + d_{dn}(v) - 2}{d_{dn}(u) + d_{dn}(v)}}$$

$$= n\sqrt{\frac{2n + (2n-1) - 2}{2n + (2n-1)}} + n\sqrt{\frac{(2n-1) + (2n-1) - 2}{(2n-1) + (2n-1)}} + n\sqrt{\frac{(2n-1) + 0 - 2}{(2n-1) + 0}}$$

$$= n\sqrt{\frac{(4n-3)}{(4n-1)}} + n\sqrt{\frac{2(n-1)}{(2n-1)}} + n\sqrt{\frac{(2n-3)}{(2n-1)}}$$

5. Result Of Gear Graph –

A Bipartite Wheel Graph Is Formed By Taking A Wheel Graph W_n, Which Has N+1 Vertices, And Adding A New Vertex Between Every Pair Of Adjacent Vertices On The Outer Rim. This Resulting Graph Is Represented By G_n And Is Also Referred To As A Gear Graph. Clearly, $|V(G_n)| = 2n+1$ And $|E(G_n)| = 3n$. A Gear Graph Gn Is Depicted In Figure 3.

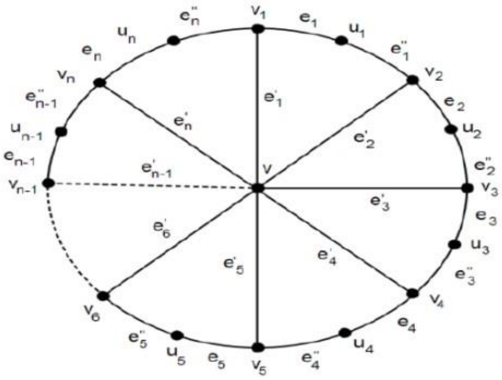

Figure 3. Gear Graph G_n

Let G_n Be A Gear Graph With 2n+1 Vertices, 3n Edges, N≥4. Then G_n Has Two Kinds Of The Downhill Degree Of Edges As Follows [8]:

$$E_1 = \{Uv \in E(G_n) |\, D_{Dn}(U) = 2n,\ D_{Dn}(V) = 2\}, |E_1| = N$$

$$E_2 = \{Uv \in E(G_n) |\, D_{Dn}(U) = 2,\ D_{Dn}(V) = 0\}, |E_2| = 2n$$

Formula 3. Let G_n Be A Gear Graph With 2n+1vertices, 3n Edges, N≥4. Then

$$ABSD(G_n) = n\sqrt{\frac{n}{(n+1)}}$$

Proof – We Obtain

$$ABSD(H_n) = \sum_{uv \in E(P_n)} \sqrt{\frac{d_{dn}(u) + d_{dn}(v) - 2}{d_{dn}(u) + d_{dn}(v)}}$$

$$= n\sqrt{\frac{2n+2-2}{2n+2}} + 2n\sqrt{\frac{2+0-2}{2+0}} = n\sqrt{\frac{n}{(n+1)}}$$

6. Conclusion

In This Paper, The Downhill Abs Index Of Standard Graphs, Wheel Graph, Helm Graph, Gear Graph Are Determined. While The Focus Is On Mathematical Calculations, The Results Obtained Will Help In

Understanding The Properties Of These Networks And For Future Research Into The Possible Applications Of These Indices.

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