

Square Sum Slither Labeling of Caterpillar Graph, Arrow Graph and Double Arrow Graph

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Abstract

Graphs have greater influence on the modern day science and technology as almost all complex structures are best explained using dots and lines. Those structures that are hard to comprehend are best explained using graphs. Labeling is one such topic in graph theory that has a gamut of applications in wide galaxy of modern science. In this paper, the acceptance of square sum slither labeling to the caterpillar graph, arrow graph and double arrow graphs are established.

Keywords: Square sum labeling, Slither Labeling, Caterpillar graph, Arrow graph and Double Arrow graph.

1. INTRODUCTION

Labeling is one of the thought instigating topics in graph theory with a spate of labeling types been defined over a period of time. This paper deals with one such labeling type, the square sum labeling. The path graph, like labeling concept, is one of the most deliberating concepts in graph theory. Many real time situations are best explained using graphs. The motivation for the development of this research paper is the compilation of types of graph labeling by J.A. Gallian [2].

BASIC TERMINOLOGIES:

Square Sum Labeling: The square sum labeling as defined by Mitesh J. Patel and G.V. Ghodasara [3] is given as; “A graph G with p vertices and q edges is said to admit square sum labeling if there is a bijective map $h:V(G) \rightarrow \{0,1,2,..,p-1\}$ such that the induced function defined by $h^*:E(G) \rightarrow \mathbb{N}$ that is defined by $h^*(uv) = (h(u))^2 + (h(v))^2$, for every $uv \in E(G)$ is an injective map”.

Slither Labeling: The label of the vertices or edges or both done in the slither pattern is the slither labeling.

Caterpillar Graph: A caterpillar graph P_n^m considered here was defined by P. Packiavathi, S. Balamurugan, and R. B. Gnanajothi [4] is a graph that is obtained from a path on n vertices by adding x_m pendant vertices to the m^{th} vertex of the path P_n . A caterpillar graph is a tree having the property that the removal of the leaves and the incident edges leaves the path P_n called as the spine of the caterpillar.

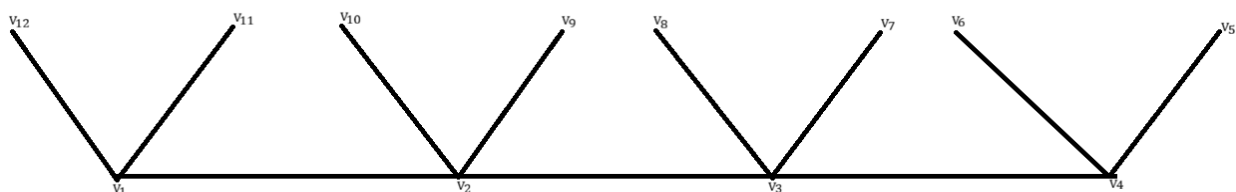


Fig 1. Caterpillar Graph P_4^2

Arrow Graph: The arrow graph AL_n is defined as follows; “An arrow graph AL_n is obtained from a ladder graph by attaching a triangle to the top end of the ladder graph. The number of vertices in the graph is $2n + 1$ and the number of edges is $3n$.”

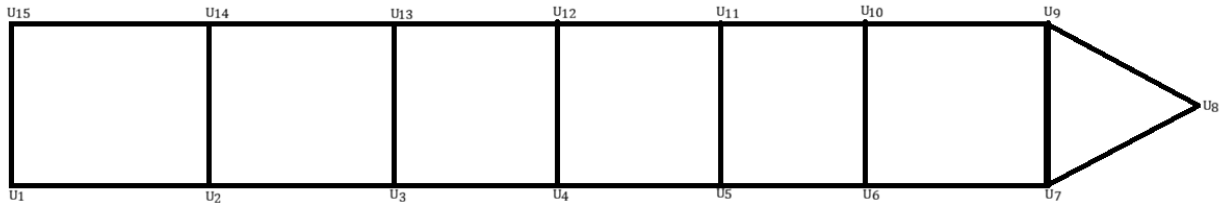


Fig 2. Arrow Graph AL_7

Double Arrow Graph: Like arrow graph, the double arrow graph is defined as follows: “An arrow graph DAL_n is obtained from a ladder graph by attaching a triangle to the top and the bottom end of the ladder graph. The number of number of vertices is $2n + 2$ and the number of edges is $3n + 2$ ”.

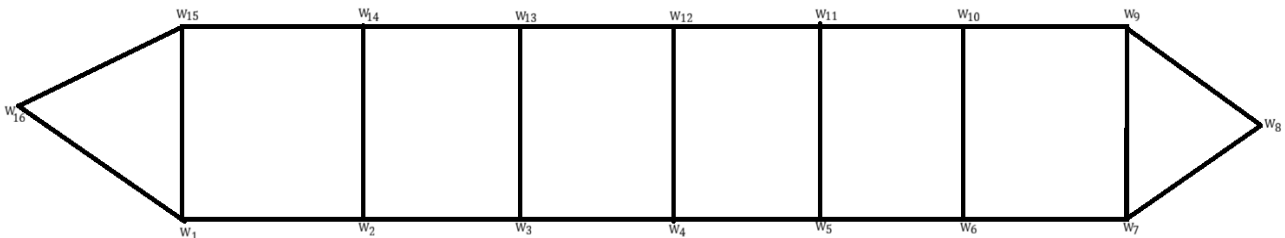


Fig 3. Double Arrow Graph DAL_7

MAIN RESULT:

Theorem 1: The Caterpillar graph P_n^m admits square sum slither labeling.

Proof: Consider the caterpillar graph formed on a path on n vertices with m leaves. Label the vertices of the graph with integers from 0 in the form of a slither pattern in such a way that the vertex labels are all distinct. The label of an edge is the sum of squares of labels of the vertices that are incident with it. Consider the graph P_6^3 , where $n = 6$ and $m = 3$ be arbitrary numbers.

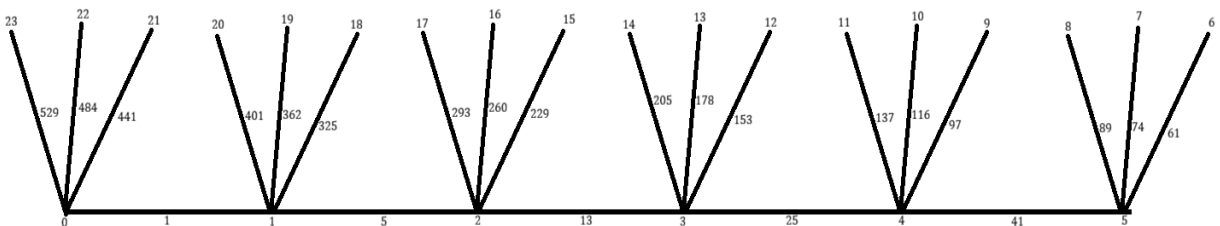


Fig 4. Square Sum Slither Labeling of Caterpillar Graph P_6^3

In the above graph, the vertices are labelled in a slither pattern and none of the vertex label recur. The label of an edge is the sum of the squares of the labels of the vertices that are incident with it and are also distinct as well and hence the graph admits square sum slither labeling. Since $n = 6$ and $m = 3$ are arbitrary, it follows that the graph P_n^m admits square sum slither labeling for all m and n .

Theorem 2: The arrow graph AL_n admits square sum slither labeling for $n \geq 2$.

Proof: Consider the arrow graph AL_n that is formed on a ladder graph on n vertices. Label the vertices of the graph in the slither form in such a way that the vertex labels are all distinct. The label of an edge is the sum of the squares of the labels of the vertices that are incident with it. Let $n = 6$ be arbitrary that the arrow

graph is formed from a ladder on 6 vertices with a triangle mounted at the top. The slither labeling of the vertices are done as follows;

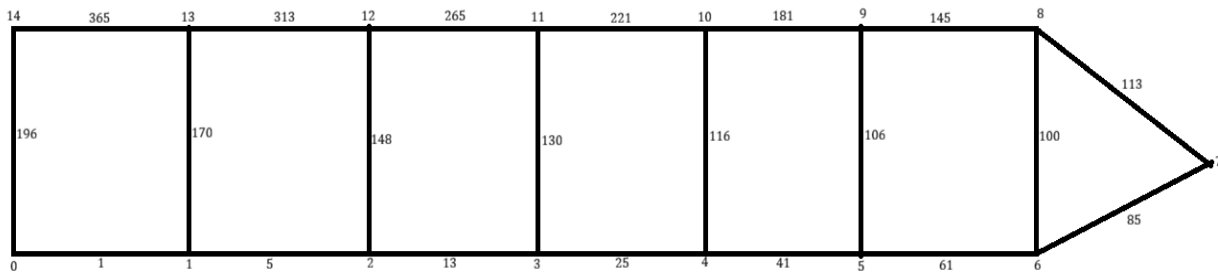


Fig 5. Square Sum Slither Labeling in Arrow Graph AL₆

In the above graph, the vertices are labelled in a slither pattern and none of the vertex labels are repeated. The label of an edge is the sum of the squares of the labels of the vertices that are incident with it and are also distinct as well and hence the graph admits square sum slither labeling. Hence the graph AL₆ admits square sum slither labeling. Since n = 6 is arbitrary, it follows that the arrow graph AL_n admits square sum slither labeling for n ≥ 2.

Theorem 3: The double arrow graph DAL_n admits square sum slither labeling for n ≥ 2.

Proof: Consider the double arrow graph DAL_n that is formed on a ladder graph on n vertices. Label the vertices of the graph in the slither form in such a way that the vertex labels are all distinct. The label of an edge is the sum of the squares of the labels of the vertices that are incident with it. Let n = 6 be arbitrary such that the double arrow graph is formed from a ladder on 6 vertices with a triangle mounted at the either side of the ladder. The slither labeling of the vertices are done as follows;



Fig 6. Square Sum Slither Labeling of Double Arrow Graph DAL₆

In the double arrow graph considered above, the vertices are labelled in a slither pattern and none of the vertex labels are repetitive. The label of an edge is the sum of the squares of the labels of the vertices that are incident with it and are also distinct as well and hence the graph admits square sum slither labeling. Hence the graph AL₆ admits square sum slither labeling. Since n = 6 is arbitrary, it follows that the arrow graph DAL_n admits square sum slither labeling for n ≥ 2.

Conclusion: In this paper, a special formation of a path graph, the caterpillar graph which is a tree, the arrow graph and the double arrow graph, both derived from ladder graphs are considered and the admittance of square sum slither labeling of the graphs is established. In a similar way, different graphs can be considered and the square sum slither labeling can be applied to them.



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