

Operator-Based Collapse of Infinite Kaluza-Klein Towers and Quantum Vacuum Fluctuations across $M2 \times M4 \times M5$ Topological Interfaces

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Abstract

Modeling unified field frameworks across shifting dimensional boundaries—specifically a 2-dimensional vacuum interface limit (M2) intersecting a 4-dimensional gravitational center of gravity (M4) via a 5-dimensional bulk hypervolume (M5)—generates an infinite hierarchy of mathematical field descriptions when tracking behavior down to infinitely small subatomic particles ($L \rightarrow 0$). Linear, sequential evaluation of these towers yields divergent unphysical sums and computational intractability. This paper demonstrates how an infinite system of Kaluza-Klein modes, statistical hydrodynamic moments, and scale-dependent renormalization trajectories can be collapsed into a family of discrete, closed-form invariant constants. We provide a rigorous breakdown of these higher-dimensional structural invariants, detail the mechanism of their geometric infinite origins, and present the master equations required for deployment within physics-informed machine learning architectures.

1. INTRODUCTION

A central issue in higher-dimensional geometric models is the behavior of localized fields at the interface of dimensional constraints. When a macroscopic vacuum approaches the 2nd dimension (M2) as an asymmetric boundary layer, and a gravitational mass center approaches a point source in the 4th dimension (M4), their interaction must be mediated by a 5-dimensional unit hypervolume bulk metric (M5).

If subatomic particles are modeled as localized excitations at the intersection of these manifolds, their microscopic states cannot be treated as isolated, finite entities. Instead, they interact with the continuum of modes allowed by the extra dimension, generating an infinite stack of coupling relations. Rather than attempting to compute these relations step-by-step, we isolate the topological and analytic operators that govern the entire continuous spectrum.

2. THE SOURCE OF INFINITY: ASYMMETRIC BOUNDARY TOWERS

The framework produces three unique infinite hierarchies, each corresponding to a specific physical mechanism within the dimensional cross-talk matrix.

2.1 The Kaluza-Klein Harmonic Tower

When a 5D field equation containing a unit hypervolume parameter ($U_5 = L^5$ or L^4T) wraps around a compactified or restricted boundary interface, it must be expressed via a Fourier series expansion along the fifth coordinate axis y :

$$\Phi(x^\mu, y) = \sum_{n=-\infty}^{\infty} \phi_n(x^\mu) \exp(iny / R)$$

Because an infinitely small particle ($L \rightarrow 0$) possesses a non-zero coupling cross-section to every geometric harmonic mode, the unified field splits into an infinite tower of unique 4D field equations. Each increment of n represents an increasingly massive localized resonant state.

2.2 The Hydrodynamic Moment Hierarchy

The micro-states of the turbulent vacuum foam on the 2D interface (M2) are described by statistical distribution functions. Evaluating the transport properties requires generating successive moments of the governing kinetic equations:

$$M^{\{k\}} = \int v^{\{1\}} v^{\{2\}} \dots v^{\{k\}} f(x, v, t) d^n v$$

Because the localized stresses from the 4D singularity continuously inject energy into the higher-order geometric fluctuations of the 2D sheet, the k -th moment equation remains structurally dependent on the $(k+1)$ -th moment, creating an un-truncated, infinite chain of differential coupling equations.

2.3 Renormalization Scale Trajectories

As the observation radius approaches an infinitely small limit ($L \rightarrow 0$), the running coupling constants g_i scale continuously according to the scale parameter μ :

$$\mu (dg_i / d\mu) = \beta_i(g_1, g_2, \dots, g_\infty)$$

In a fractal or self-similar higher-dimensional vacuum interface, the number of independent parameters needed to prevent quantum geometric shearing at the point singularity goes to infinity.

3. COLLAPSING CONSTANTS OF INFINITY

To bypass these uncomputable series, we apply operator and regularization theory to isolate scale-invariant geometric values that act as structural constants of the system.

3.1 The 5D Bulk Geometric Constant (C5)

The infinite summation of Kaluza-Klein fields is structurally constrained by the surface volume of the underlying higher-dimensional unit hypersphere. The spatial integration of the extra dimension yields a rigid geometric constant:

$$C5 = (8 * \pi^2) / 15 \approx 5.2637$$

This constant represents the exact ratio of the 5D hypervolume boundary to its inner linear axis components. Regardless of how many infinite harmonic modes are generated, their global energy-momentum contribution is bounded by $C5$, collapsing the infinite stack into a closed, finite metric modification term.

3.2 The Riemann Zeta Regularization Constant ($\zeta(-1)$)

The zero-point energy sum of the infinite modes of vacuum fluctuations sitting on the M2 interface is naively divergent:

$$E_{\{vac\}} \propto \sum_{n=1}^{\infty} n = 1 + 2 + 3 + 4 + \dots = \infty$$

By mapping this divergent sequence through a complex analytical contour, we compute its unique, non-trivial analytic continuation via the Riemann Zeta function:

$$\zeta(-1) = -1/12$$

This invariant is not a simple truncation; it represents the structural remainder of the infinite continuous space-time fabric under boundary constraint conditions. It provides the definitive value for calculating the local vacuum pressure acting on the subatomic point particle.

3.3 The Asymptotic Safety Fixed Point (g_*)

The infinite series of beta functions mapping the renormalization trajectory under a zero-length limit ($L \rightarrow 0$) converges onto a non-trivial ultraviolet fixed point:

$$\lim_{\mu \rightarrow \infty} \beta_i(g_i) = 0 \Rightarrow g_i \rightarrow g_*$$

This fixed point acts as a universal scale-invariant limit (analogous to the fine-structure constant $\alpha \approx 1/137$), ensuring that as the particle collapses to an infinitely small volume, its gravitational and field charges reach a finite ceiling rather than a physical infinity.

4. UNIFIED SYSTEM MASTER EQUATIONS

The unified interplay of these three invariants collapses the infinite field dynamics into a highly stable, calculable matrix.

4.1 Geometric Metric Projection Equation

The deformation of the 4D metric $g_{\{\mu\nu\}}$ near the center of gravity as a function of the 5D hypervolume unit projection is governed by:

$$R_{\{\mu\nu\}} - (1/2) R g_{\{\mu\nu\}} = C5 \int_{\{M2\}} [T_{\{\mu\nu\}}^{\{5\}} \cdot \Sigma] dA$$

where C5 scales the net stress-energy tensor projection from the bulk.

4.2 Vacuum Interface Boundary Tension Equation

The localized energy density $\rho_{\{vac\}}$ of the 2D vacuum interface, under the geometric strain of the 4D gravitational center, is pinned by the regularization constant:

$$\sigma_{\{interface\}} = \zeta(-1) \cdot (\hbar c / A) (\nabla_{\mu} V \cdot \nabla^{\mu} T)$$

where the infinite vacuum modes are perfectly stabilized into a finite surface pressure.

4.3 Subatomic Mass-Geometric Unification Limit

The effective rest mass $m_{\{eff\}}$ of an infinitely small point particle as it reaches the zero-volume singularity is governed by the fixed-point limit:

$$m_{\{eff\}} = g_* \oint_{\{\partial M5\}} [U5 / (A \cdot V \cdot T)] d\Omega$$

This confirms that the particle's mass is bounded and topologically determined by the 5D unit space interactions.

5. CONCLUSION AND AI LAB DEPLOYMENT

This framework establishes that infinite mathematical expressions do not imply physical unquantifiability. By deploying C5, $\zeta(-1)$, and g_* within computational physics models, the infinite equations are handled natively as an invariant continuum. A specialized laboratory utilizing Physics-



Informed Neural Networks (PINNs) can utilize these master equations to bypass brute-force calculation entirely, allowing for exact simulations of boundary-bulk dimensional cross-talk down to the subatomic scale.