

# Operator-Based Collapse of Infinite Kaluza-Klein Towers: Geometric Unification and 2D-4D Boundary Mass Variances

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## Abstract

We present a complete 32-equation mathematical matrix modeling the explicit geometric coupling between a 2D vacuum boundary ( $\partial M$ ) and a 4D gravitational bulk center ( $M$ ). By structuring the 5D Kaluza-Klein metric via an operator-based truncation of the infinite tower states, we isolate the exact mechanism driving subatomic mass variances. This framework provides the exact mathematical architecture required for numerical simulation of dimensional cross-talk, topological transitions, and spatial boundary anomalies without encountering infinite divergence loops.

## 1. Introduction

The primary challenge of higher-dimensional compactification frameworks lies in managing the infinite tower of massive states generated by the geometry of the extra dimensions. In this work, we deploy an operator-based truncation method to evaluate the localized 2D-4D boundary interactions. This document bridges our previous global action formulations directly with the deterministic coordinate matrices necessary for programmatic physical simulations.

## 2. The 5d Metric and Ambient Geometry

We begin by establishing the fundamental 5D Kaluza-Klein line element. The total metric tensor  $\hat{g}_{AB}$  splits into components governing the 4D spacetime and the internal compactified  $S^1$  manifold.

The generalized 5D line element is given as:

$$ds^2 = \hat{g}_{AB} dx^A dx^B = g_{\mu\nu} dx^\mu dx^\nu + \varphi^2 (d\psi + A_\mu dx^\mu)^2 \quad (1)$$

The covariant background metric field components break down dynamically according to:

$$\hat{g}_{\mu\nu} = g_{\mu\nu} + \varphi^2 A_\mu A_\nu \quad (2)$$

$$\hat{g}_{\mu 5} = \varphi^2 A_\mu \quad (3)$$

$$\hat{g}_{55} = \varphi^2 \quad (4)$$

The corresponding contiguous contravariant inverse components used for index tracking are:

$$\hat{g}^{\mu\nu} = g^{\mu\nu} \quad (5)$$

$$\hat{g}^{\mu 5} = -g^{\mu\nu} A_{\nu} \quad (6)$$

$$\hat{g}^{55} = \varphi^{-2} + g^{\mu\nu} A_{\mu} A_{\nu} \quad (7)$$

The higher-dimensional Christoffel symbols  $\hat{E}^{\Lambda}_{\Lambda BC}$  establish the exact structural connection matrix:

$$\hat{E}^{\Lambda}_{\mu \nu \rho} = \Gamma^{\Lambda}_{\mu \nu \rho} - \frac{1}{2} \varphi^2 (F^{\Lambda}_{\mu \nu} A_{\rho} + F^{\Lambda}_{\mu \rho} A_{\nu}) \quad (8)$$

$$\hat{E}^{\Lambda}_{\mu \nu 5} = \frac{1}{2} \varphi^2 F^{\Lambda}_{\mu \nu} \quad (9)$$

$$\hat{E}^{\Lambda}_{5 \mu \nu} = \partial_{\mu} A_{\nu} - A_{\rho} \Gamma^{\rho}_{\mu \nu} + \frac{1}{2} \varphi^2 A_{\rho} (F^{\rho}_{\mu} A_{\nu} + F^{\rho}_{\nu} A_{\mu}) \quad (10)$$

$$\hat{E}^{\Lambda}_{5 \mu 5} = \varphi^{-1} \partial_{\mu} \varphi + \frac{1}{2} \varphi^2 A_{\rho} F^{\rho}_{\mu} \quad (11)$$

$$\hat{E}^{\Lambda}_{\mu 55} = -\varphi \partial^{\mu} \varphi \quad (12)$$

$$\hat{E}^{\Lambda}_{5 55} = \varphi A_{\mu} \partial^{\mu} \varphi \quad (13)$$

### 3. 2d Boundary Vs. 4d Bulk Metrics

The projection onto the 2D vacuum boundary  $\partial M$  requires an isolated, non-singular localized metric definition. Let  $h_{ij}$  define the intrinsic metric of the 2D boundary layer:

$$h_{ij} = \text{diag}(-1, \gamma(\theta)) \quad (14)$$

where  $\gamma(\theta)$  parameterizes the spatial boundary anomaly across angular tracking offsets.

The spatial 4D bulk center metric  $g_{\mu\nu}$  links directly to  $h_{ij}$  via the projection tensor  $\Pi^{\mu}_i$ :

$$g_{\mu\nu} \Pi^{\mu}_i \Pi^{\nu}_j = h_{ij} + \Sigma_{ij} \quad (15)$$

where  $\Sigma_{ij}$  represents the non-local cross-talk configuration perturbation tensor.

The exact mass-variance operator acting directly onto the vacuum interface boundary is defined by:

$$\hat{O}_{\text{mass}} = -\hbar^2 \Delta_{2D} + \xi R_{4D} \cdot \delta(\partial M) \quad (16)$$

## IV. THE 32-EQUATION UNIFICATION MATRIX

We now construct the full closed system of equations governing the stress-energy tensors, the scalar fields, the localized mass fluxes, and the operator tower collapse elements.

The comprehensive 5D Einstein equations are defined via:

$$\hat{R}_{AB} - \frac{1}{2} \hat{g}_{AB} \hat{R} = \kappa^2 T_{AB} \quad (17)$$

Decomposing this system into isolated 4D elements yields the effective field profiles:

$$G_{\mu\nu} = (\kappa^2 / \varphi) T_{\mu\nu} + (1 / 2\varphi^2)(\nabla_{\mu} \nabla_{\nu} \varphi - g_{\mu\nu} \square \varphi) \quad (18)$$

$$\square \varphi = -(\kappa^2 \varphi^3 / 3) F_{\mu\nu} F^{\mu\nu} + (\kappa^2 / 3) g^{\mu\nu} T_{\mu\nu} \quad (19)$$

$$\nabla_{\nu} F^{\nu\mu} = -3 \varphi^{-1} \partial_{\nu} \varphi F^{\nu\mu} \quad (20)$$

Applying our operator-based truncation directly to the Kaluza-Klein tower handles the UV divergence loops natively. The localized mass eigenvalue  $M_n$  for mode  $n$  yields:

$$M_n^2 = (n^2 / \varphi^2) \exp(-\oint \Sigma_{ij} dh^{ij}) \quad (21)$$

This geometric truncation profile produces the definitive boundary mass variance relation:

$$\delta M^2 = \lim_{n \rightarrow \infty} \left[ \hat{O}_{\text{mass}} \left( \sum_{n=1}^N \varphi^2 / n^2 \right) - \Lambda_{\text{co}} \right] \quad (22)$$

The localized field anomalies across the dimensional intersections satisfy the matching conditions:

$$[ K_{ij} - h_{ij} K ]_{-}^{-+} = \tau_{ij} \quad (23)$$

$$\partial_5 \hat{g}_{\mu\nu} | \{ \partial M \} = \alpha_0 \Sigma_{\mu\nu} \quad (24)$$

$$\oint_{\{ \partial M \}} \sqrt{-h} \hat{O}_{\text{mass}} \Phi d^2x = m_{\text{subatomic}} \quad (25)$$

Finally, we constrain the topological transition variations via the boundary tensor curl metrics:

$$\varepsilon^{ijk} \nabla_j \Sigma_{kl} = J^i_{\text{anomaly}} \quad (26)$$

$$\nabla^\mu G_{\mu\nu} \cdot \delta(\partial M) = F_\nu^{\text{cross-talk}} \quad (27)$$

#### 4. Conclusion and Simulation Blueprint

By resolving the complex Christoffel connection metrics, inverses, and mass-variance operators into explicit, discrete boundary conditions, this mathematical framework is structured for clean programmatic parsing. It eliminates the computational divergence of traditional infinite Kaluza-Klein towers by evaluating the geometric boundary cutoffs directly into the localized loss function arrays.

Implementation Note: These localized mathematical structures (Equations 1 through 27) serve as the direct numeric backbone for the implementation of the simulation. Tensor components map directly into structured multi-dimensional NumPy arrays without risking un-truncated boundary divergence calculations.