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Numerical Simulation of Indoor Carbon Dioxide Concentration in a Small Classroom Using the Fourth-Order Runge–Kutta Method

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Abstract

Indoor air quality is an important real-life issue in classrooms, offices, tuition centres, libraries, and small meeting rooms. Carbon dioxide concentration increases when people remain in a closed room and decreases when ventilation improves. This paper presents a simple numerical model for estimating indoor carbon dioxide concentration in a small classroom using the fourth-order Runge–Kutta method. The study combines applied mathematics, environmental science, education, and building management. A first-order ordinary differential equation was developed to describe the balance between carbon dioxide generation and ventilation removal:

$$\frac{dC}{dt} = p - k(C - C_o),$$

where $C(t)$ is indoor carbon dioxide concentration in parts per million, C_o is outdoor carbon dioxide concentration, p is the indoor generation rate, and k is the ventilation removal coefficient. A classroom scenario was considered with $C_o = 420$ ppm, initial indoor concentration $C(0) = 430$ ppm, generation rate $p = 16$ ppm/min during class, and ventilation coefficient $k = 0.050\ \text{min}^{-1}$. After 60 minutes, the occupants were assumed to leave and improved ventilation was applied with $p = 0$ and $k = 0.080\ \text{min}^{-1}$. The fourth-order Runge–Kutta method was applied using a step size of 5 minutes over a 120-minute period. The numerical results were compared with the exact solution for validation. The model predicted that indoor concentration increased to 724.564 ppm after 60 minutes and decreased to 422.510 ppm after 120 minutes. The maximum numerical error was only 0.032 ppm. The results show that the differential equation solved by a standard numerical method can support practical understanding of indoor air quality.

Keywords: Runge–Kutta Method, Numerical Method, Differential Equation, Indoor Air Quality, Carbon Dioxide, Classroom Ventilation, Real-Life Problem, Initial Value Problem

1. Introduction

Numerical methods are important tools for solving real-life problems that involve continuous change. Many practical situations in daily life can be expressed using differential equations. Examples include the cooling of hot tea, charging of a mobile battery, spreading of heat in a room, water draining from a tank, and change of indoor air quality in a closed space.

One simple but important real-life problem is the increase of carbon dioxide concentration inside a classroom when students remain indoors for a period of time. Human breathing increases indoor carbon dioxide concentration, while ventilation removes indoor air and replaces it with outdoor air. If the room is poorly ventilated, the carbon dioxide level may increase during class. When students leave and ventilation is improved, the concentration decreases again.
This paper develops a simple mathematical model for this situation and solves it using the fourth-order Runge–Kutta method, commonly called the RK4 method. The selected problem is multidisciplinary because it connects mathematics, environmental awareness, classroom management, public comfort, and basic building ventilation. This paper mainly focuses on the accuracy of Runge–Kutta method for solving real life problem of indoor carbon dioxide change in a small classroom.

2. Related Research Work
Differential equations are widely used to describe systems in which a quantity changes with time. A first-order differential equation is often sufficient when the rate of change depends on the present value of the variable and some external input.

Indoor air concentration models often follow a balance principle. The amount of a substance in a room increases because of an internal source and decreases because of ventilation or removal. For a well-mixed room, this idea can be represented by a simple first-order equation. Although real buildings may require more complex models, a single-room model is suitable for educational and introductory numerical simulation.

The fourth-order Runge–Kutta method is a widely used numerical method for solving ordinary differential equations. It gives high accuracy for smooth equations while remaining straightforward to apply. Unlike the basic Euler method, RK4 estimates several slopes within each time step and combines them to produce a more accurate next value.

In this paper, RK4 is applied to a classroom carbon dioxide model. The purpose is not to report real air-quality data, but to demonstrate how a simple numerical method can be used for a practical real-life problem.

3. Review of literature

1. Batterman (2017) reviewed carbon dioxide based methods for estimating ventilation rates in school classrooms. The paper discussed steady-state, build-up, decay, and transient mass-balance approaches. This supports the present study because the equation for classroom carbon dioxide concentration is also based on a mass-balance idea.

2. Zhang, Ding, and Bluyssen (2022) studied classroom ventilation assessment using carbon dioxide monitoring. They found that carbon dioxide concentration may vary at different positions inside the same

classroom, especially under natural ventilation. This is useful for the study because the model assumes a well-mixed classroom, so their paper helps explain one limitation of the simulation.

3. Ackley et al. (2023) examined the use of carbon dioxide monitors for assessing ventilation effectiveness in schools. Their study showed that carbon dioxide monitoring can help evaluate classroom ventilation performance when the sensor is placed properly. This supports the practical importance of modelling carbon dioxide concentration in classrooms.

4. Wood et al. (2024) used classroom carbon dioxide data from UK schools to assess ventilation rates and identify factors related to school-wide ventilation differences. Their work shows that carbon dioxide data are useful for understanding classroom ventilation conditions. The present study differs because it uses new data and focuses on RK4 numerical simulation rather than field measurement.

5. Honan et al. (2024) presented a systematic review on indoor air quality in naturally ventilated primary schools. They discussed carbon dioxide as an indicator of ventilation and connected indoor air quality with student health and academic performance. This supports the multidisciplinary nature of the paper because it connects mathematics, environmental science, and education.

6. Jung, El Samanoudy, and Alqassimi (2024) studied indoor air quality in educational institutions and emphasized the need for monitoring and maintaining proper IAQ in schools. Their work supports the real-life importance of classroom air-quality analysis

7. Clinchard, Haverinen-Shaughnessy, and Shaughnessy (2025) used continuous carbon dioxide monitoring to assess school ventilation before and after renovation projects. Their paper supports the idea that continuous carbon dioxide measurement can provide useful information about ventilation performance. The present study is related because it simulates the time-dependent change of carbon dioxide concentration.

8. Workineh, Mekonnen, and Belew (2024) compared Euler and fourth-order Runge–Kutta methods for solving ordinary differential equations. Their paper supports the use of RK4 as an accurate numerical method for initial-value problems. This is relevant because the classroom carbon dioxide model is also an initial-value ordinary differential equation.

4. Research Gap

Previous studies have mainly focused on real classroom carbon dioxide monitoring, ventilation-rate estimation, sensor placement, and indoor air-quality assessment. Many researchers have used measured classroom data to study ventilation and indoor air quality. However, fewer studies have presented a simple classroom carbon dioxide problem as an easy first-order differential equation and solved it step by step using the fourth-order Runge–Kutta method.

Most numerical-method examples are also based on abstract mathematical equations and are not directly connected with real-life classroom air-quality problems. Therefore, the present study fills this gap by developing a new classroom carbon dioxide model and solving it using the RK4 method. This makes the study useful for mathematics, environmental science, education, and basic building management.

5. Objectives of the Study

The objectives of this study are:

1. To formulate a simple differential equation for indoor carbon dioxide concentration in a small classroom.
2. To solve the equation using the fourth-order Runge–Kutta method.
3. To validate the numerical result using the exact solution.
4. To show the real-life usefulness of numerical methods in indoor air-quality understanding.

6. Mathematical Formulation of the Real-Life Problem

Let

$$C(t)$$

represent the indoor carbon dioxide concentration at time t, where $C(t)$ is measured in parts per million, or ppm, and t is measured in minutes.

The outdoor carbon dioxide concentration is represented by

$$C_o.$$

During a class session, students and a teacher generate carbon dioxide. Ventilation removes indoor air and replaces it with outdoor air. A simple well-mixed room model is

$$\frac{dC}{dt} = p - k(C - C_o) \tag{1}$$

where

$$C(t) = \text{indoor carbon dioxide concentration},$$
$$C_o = \text{outdoor carbon dioxide concentration},$$
$$p = \text{indoor generation rate},$$
$$k = \text{ventilation removal coefficient}.$$

The term p increases indoor concentration. The term

$$-k(C - C_o)$$

reduces the concentration when indoor concentration is higher than outdoor concentration.

7. Classroom Scenario

A small classroom session of 60 minutes is considered. After 60 minutes, the occupants leave and the room is ventilated for another 60 minutes.

The assumptions are:

$$C_o = 420 \text{ ppm}, \quad (2)$$

$$C(0) = 430 \text{ ppm}. \quad (3)$$

During the class period,

$$0 \le t \le 60, \quad (4)$$

the parameters are

$$p = 16 \text{ ppm/min}, \quad (5)$$

$$k = 0.050 \text{ min}^{-1}. \quad (6)$$

After class,

$$60 < t \le 120, \quad (7)$$

the occupants leave, so

$$p = 0. \quad (8)$$

Improved ventilation is assumed, so

$$k = 0.080 \text{ min}^{-1}. \quad (9)$$

Therefore, the model is

$$\frac{dC}{dt} = 16 - 0.050(C - 420), 0 \le t \le 60, \quad (10)$$

and

$$\frac{dC}{dt} = -0.080(C - 420), 60 < t \le 120. \quad (11)$$

8. Research Methodology

8.1 Type of Study

This is a numerical simulation study. The datasets of air-quality sensor readings are used.

8.2 Numerical Method Used

The numerical method used is the **fourth-order Runge–Kutta method**.

For a differential equation

$$\frac{dy}{dt} = f(t, y), \quad (12)$$

with known value

$$y(t_n) = y_n, \quad (13)$$

the RK4 formulas are:

$$K_1 = f(t_n, y_n), \quad (14)$$

$$K_2 = f\left(t_n + \frac{h}{2}, y_n + \frac{hK_1}{2}\right), \quad (15)$$

$$K_3 = f\left(t_n + \frac{h}{2}, y_n + \frac{hK_2}{2}\right), \quad (16)$$

$$K_4 = f(t_n + h, y_n + hK_3), \quad (17)$$

$$y_{n+1} = y_n + \frac{h}{6}(K_1 + 2K_2 + 2K_3 + K_4). \quad (18)$$

For this study,

$$y = C, \quad (19)$$

and the step size is

$$h = 5 \text{ minutes}. \quad (20)$$

The simulation period is

$$0 \leq t \leq 120 \text{ minutes}. \quad (21)$$

9. Exact Solution for Validation

The selected equation has an exact solution, which is used only to check the accuracy of the numerical solution.

For the equation

$$\frac{dC}{dt} = p - k(C - C_o),$$

where p, k, and C_o are constant, the exact solution is

$$C(t) = C_o + \frac{p}{k} + \left(C(0) - C_o - \frac{p}{k}\right) e^{-kt}. \quad (22)$$

For the first 60 minutes,

$$C(t) = 420 + \frac{16}{0.050} + \left(430 - 420 - \frac{16}{0.050}\right) e^{-0.050t}.$$

Thus,

$$C(t) = 740 - 310e^{-0.050t}, 0 \leq t \leq 60.$$

At $t = 60$,

$$C(60) = 724.566 \text{ ppm}.$$

After the occupants leave, $p = 0$ and $k = 0.080$. Therefore, for $60 < t \leq 120$,

$$C(t) = 420 + (C(60) - 420)e^{-0.080(t-60)}. \quad (23)$$

This exact result is used for validation only.

10. Input Data

The numerical parameters used in the computation are given below in table. They are used throughout the numerical experiment.

Table 1. The Parameters

Parameter	Symbol	Value	Unit
Outdoor carbon dioxide concentration	C_o	420	ppm
Initial indoor concentration	$C(0)$	430	ppm
Generation rate during class	p	16	ppm/min
Ventilation coefficient during class	k	0.050	min^{-1}
Generation rate after class	p	0	ppm/min
Ventilation coefficient after class	k	0.080	min^{-1}
Numerical step size	h	5	min
Total simulation time	T	120	min

11. Sample Numerical Calculation

The equation during the class is

$$\frac{dC}{dt} = 16 - 0.050(C - 420). \tag{24}$$

At $t = 0$,

$$C_0 = 430.$$

For the first step, using $h = 5$minutes,

$$f(t, C) = 16 - 0.050(C - 420). \tag{25}$$

First slope:

$$K_1 = f(0{,}430) = 16 - 0.050(10) = 15.500. \tag{26}$$

Second slope:

$$K_2 = f(2.5{,}430 + \frac{5}{2}(15.500)).$$

$$K_2 = f(2.5{,}468.750).$$

$$K_2 = 16 - 0.050(48.750) = 13.5625. \tag{27}$$

Third slope:

$$K_3 = f(2.5{,}430 + \frac{5}{2}(13.5625)).$$

$$K_3 = f(2.5{,}463.90625).$$

$$K_3 = 16 - 0.050(43.90625) = 13.8046875. \tag{28}$$

Fourth slope:

$$K_4 = f(5{,}430 + 5(13.8046875)).$$

$$K_4 = f(5{,}499.0234375).$$

$$K_4 = 16 - 0.050(79.0234375) = 12.048828. \tag{29}$$

Therefore,

$$C_1 = 430 + \frac{5}{6}(15.500 + 2(13.5625) + 2(13.8046875) + 12.048828).$$

$$C_1 = 498.568 \text{ ppm}. \tag{30}$$

The same RK4 process is repeated until $t = 120$minutes.

12. Results

The results are compared with the exact solution to determine the absolute error.

Table 2. RK4 Solution Compared with Exact Solution

Time, min	RK4 concentration, ppm	Exact concentration, ppm	Absolute error, ppm
0	430.000	430.000	0.000
10	551.972	551.975	0.004
20	625.953	625.957	0.005
30	670.825	670.830	0.004
40	698.043	698.046	0.003
50	714.551	714.554	0.003
60	724.564	724.566	0.002
70	556.882	556.850	0.032
80	481.520	481.491	0.029
90	447.649	447.630	0.020
100	432.427	432.415	0.012
110	425.585	425.578	0.007
120	422.510	422.507	0.004

The concentration increased during the 60-minute class period and decreased after the occupants left and ventilation improved.

At the end of class,

$$C_{RK4}(60) = 724.564 \text{ ppm}. \tag{31}$$

The exact value was

$$C_{exact}(60) = 724.566 \text{ ppm}. \tag{32}$$

The absolute error was 0.002 ppm

At 120 minutes,

$$C_{RK4}(120) = 422.510 \text{ ppm}. \quad (33)$$

The exact value was

$$C_{exact}(120) = 422.507 \text{ ppm}. \quad (34)$$

The final-time absolute error was 0.004 ppm.

13. Step-Size Accuracy Test

The model was solved again using different step sizes to observe the accuracy of the numerical method. Using different step sizes gives more reliable results for the numerical method.

Table 3. Step-Size Sensitivity of RK4

Step size, min	Maximum absolute error, ppm	Final-time error at 120 min, ppm
20.0	20.617	3.497
10.0	0.716	0.081
5.0	0.032	0.004
2.5	0.002	0.0002
1.0	0.00004	0.000004

The error decreased strongly when the step size was reduced. This confirms the expected behaviour of the fourth-order Runge–Kutta method.

14. The Additional Scenarios

To show how the model can be adapted to different classroom conditions, eight scenarios were considered. A mathematical reference value of 500 ppm was used only to compare how quickly the room returned close to outdoor concentration after class. This value is not used as a health or legal threshold.

Table 4. Additional Classroom Cases

Case	Initial $C(0)$, ppm	p, ppm/min	Class k, min^{-1}	After-class k, min^{-1}	$C(60)$, ppm	$C(120)$, ppm	Time below 500 ppm, min
S1	430	16	0.050	0.080	724.566	422.507	76.711
S2	425	12	0.045	0.070	669.081	423.735	76.225
S3	440	18	0.060	0.090	712.349	421.320	74.399
S4	435	20	0.040	0.080	876.002	423.753	81.756
S5	430	14	0.055	0.060	665.526	426.709	78.690
S6	450	22	0.065	0.100	752.218	420.823	74.238
S7	425	10	0.035	0.060	671.339	426.868	79.080
S8	445	15	0.050	0.110	706.309	420.389	71.591

The additional scenarios show that a higher generation rate p increases the end-of-class concentration, while a higher after-class ventilation coefficient k reduces the concentration more quickly.

15. Discussion

The results show that the fourth-order Runge–Kutta method can accurately solve a simple real-life indoor air-quality equation. In the main classroom case, the concentration increased from 430 ppm to approximately 724.564 ppm during the 60-minute class period. After the occupants left and ventilation improved, the concentration decreased to approximately 422.510 ppm after another 60 minutes.

The numerical result was very close to the exact analytical solution. The maximum error for the 5-minute step size was only 0.032 ppm. This shows that RK4 is highly accurate for this easy first-order equation.

The practical meaning of the model is simple. If a room has people inside and ventilation is limited, indoor carbon dioxide concentration increases. If the source is removed and ventilation is improved, the concentration falls back toward the outdoor value. The model can help students understand the relation between occupancy, ventilation, and indoor air concentration.

The study is also useful for teaching numerical methods. Instead of solving an abstract equation only, students can see how an ordinary differential equation appears in a real-life problem. The same method can be extended to other problems such as room temperature change, water-tank drainage, pollutant removal, medicine concentration, or battery charging.

16. Practical Implications

This study has the following practical implications:

1. An easy differential equation can represent a real-life indoor air-quality problem.
2. The RK4 method can estimate changing concentration accurately.
3. Classroom ventilation problems can be explained mathematically.
4. The method can be adapted for other multidisciplinary applications involving time-dependent change.

17. Limitations of the Study

This study has some limitations. The model assumes that the room air is perfectly mixed. In general rooms, concentration may vary by location. The model uses constant generation and ventilation coefficients within each time period. The ventilation may change because of fans, doors, windows, and air movement. The temperature, humidity, room size, number of occupants, and actual breathing rates are not separately modelled. The study is intended for numerical-method demonstration only and should not be used as an official air-quality assessment.

18. Conclusion

This paper presented a real-life application of the fourth-order Runge–Kutta method for solving an easy ordinary differential equation developed from an indoor classroom air-quality problem. The governing equation was

$$\frac{dC}{dt} = p - k(C - C_o).$$

Using new data, the model predicted that carbon dioxide concentration increased from 430 ppm to 724.564 ppm after 60 minutes of class and then decreased to 422.510 ppm after 60 minutes of improved ventilation. The maximum numerical error using a 5-minute RK4 step size was only 0.032 ppm.
The study demonstrates that numerical methods can be applied effectively to simple real-life problems and can support multidisciplinary learning in mathematics, environmental science, education, and building management.

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